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References

Lane, B. H., 1935, Suggestions to authors of papers submitted for publication by the United States Geological Survey: U. S. Geol. Survey, Washington, D. C. (For sale by Superintendent of Documents, Washington, D. C., 85c.)

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AUGUST 1953

AN EXPERIMENTAL TEST OF VISUAL COMPARISON TECHNIQUE IN ESTIMATING TWO DIMENSIONAL SPHERICITY AND ROUNDNESS OF QUARTZ GRAINS

MELVIN A. ROSENFELD AND JOHN C. GRIFFITHS

ABSTRACT. This experiment attempts to evaluate the visual comparison technique for the estimation of sphericity and roundness of quartz grains in sediments. The experiment was designed to enable the data to be statistically analyzed by means of analysis of variance—a powerful statistical device which has a wide field of application in geology.

The results of the experiment indicate that valid estimates of sphericity and roundness may be obtained by visual comparison with standards; but unless the variation due to differences between operators is suitably evaluated, visually estimated differences in sphericity and roundness cannot be attributed solely to differences between grains but are affected to an unknown degree by operator variation. The sympathetic relationship between sphericity and roundness, for example, appears on the basis of this experiment to arise from psychological bias on the part of operators and not from an inherent relationship between these properties of quartz grains.

INTRODUCTION

IN attempting to estimate quantitatively the variation in the properties of sediments it is essential to evaluate the limitations of technique. Certain fundamentals must be fulfilled before a technique may be adopted as a routine for estimating variation in some particular property, and in testing the technique, the more restricted the test the more restricted the conclusions to be drawn from it. Ordinarily an experiment which purports to compile a set of measured data is somewhat circumscribed in its implications unless an extensive background of similarly measured reference data is available to which to relate the new experimental determinations. Before attempting to compile the extensive background it is, however, necessary to choose a suitable, efficient technique which will lead to meaningful measurements. Hence painstakingly thorough evaluation of

a technique is essential, and if this aspect is neglected it will inexorably lead to very considerable wasted effort.

In the present case we are interested in attempting to evaluate limitations in the estimation of sphericity and roundness. The most rapid technique for both purposes and the only practicable technique for routine estimation of roundness of large numbers of grains is by means of visual comparison charts (Krumbein, 1941*a*, and Rittenhouse, 1943), and if this approach could be safely adopted, a large amount of data on different sediments could be compiled in a reasonably short space of time.

Initially it is clear both from laboratory experiments (Krumbein, 1941*b*) and actual measurements of quartz grains in sediments (Pettijohn, 1949, p. 46-55) that the magnitude of variation measured on the conventional scale of 0-1 is likely to be very much less in the case of sphericity than roundness. On this basis, therefore, it is necessary to use greater precision in the case of estimation of sphericity than roundness and it may well be that two different techniques will be finally necessary. From the many experiments quoted in the literature (see Pettijohn, 1949, for extensive bibliography) it is also clear, and this is very fortunate, that it is much simpler to measure sphericity than roundness.

Again it is reasonably clear from Pettijohn's graphic portrayal (1949, fig. 26, p. 54) that while the trends of relationship between sphericity and roundness are similar in different sediments differences between techniques cause as much displacement in the position of the trends as differences between sediments.

Finally, we have learned by way of extensive experience that differences between operators performing the same tasks may well interfere with, and sometimes exceed, differences between sediments and/or techniques.

We may conclude, therefore, that in order to test a technique, an experiment should be reasonably comprehensive and, as the description of any technique in the literature is presumably published with the intention that other investigators should adopt it, differences between investigators should always be evaluated in testing a technique. It may not be out of place here to emphasize that reproducibility as frequently reported can be very misleading; for example, many chemical analysts will claim

quite justifiably that they can achieve duplicate or replicate analyses within enviable margins of precision, yet differences between analysts will far exceed this precision error and lead to very embarrassing results if not realized and evaluated before the compilation of the necessary extensive background of reference data (e.g., Fairbairn, 1951).

The present experiment is one of a series in which various techniques and various measurable properties of sediments are being tested with a view to ultimate adoption of suitable analytical procedures for compiling the necessary extensive background of reference data on the magnitude of variation in the properties of sedimentary rocks.

PROCEDURE AND DESIGN OF EXPERIMENT

The details of the procedure used in the estimation of variation in sphericity and roundness in quartz grains from sedimentary rocks by visual comparison with standard reference charts are adequately described in the original articles (Krumbein, 1941*a*, and Rittenhouse, 1943). In the present case 21 grains of quartz were randomly selected from a beach sand and mounted in semi-permanent mounts; the 21 grains encompassed a variation of from 0.71 to 0.93 in sphericity and from 0.10 to 0.80 in roundness.

The published charts (see literature cited) were photographically reproduced and brought to a standard size of 4″ x 5″ overall. Seven operators, all graduate students in the Division of Mineralogy of the Pennsylvania State College, took part in the test[1] which was carried out in 1948 under the immediate supervision of one of us (M.A.R.). The experiment was initially designed to evaluate differences between grains and differences between operators but, by arranging the replicate runs of each operator on different days, variation from day to day was also included. In terms of experimental design the differences between grains, differences between operators and differences between days are termed main effects, and subsidiary factors comprise three first order interactions (Youden, 1951), namely, operators and days, operators and grains, and grains and days. Finally, a second order interaction of operators and grains and days completes the list of effects included in the experiment.

[1] The seven operators were C. T. Bressler, E. Cressman, D. B. Doan, R. L. Folk, M. A. Rosenfeld, A. Swineford, and C. E. Weaver.

Table I. Mathematical model of analysis of variance for testing sphericity and roundness data

Source of variation	Degrees of freedom	Sum of squares	Mean square	Components of variance
Grains G	$g-1$	$\sum_{1}^{g}\left(\sum_{1}^{dn} x\right)^2/dn - CT$	$\frac{SS_G}{(g-1)}$	$s^2_e + ds^2_{GN} + ns^2_{DG} + nds^2_G$
Operators N	$n-1$	$\sum_{1}^{n}\left(\sum_{1}^{dg} x\right)^2/dg - CT$	$\frac{SS_N}{(n-1)}$	$s^2_e + ds^2_{GN} + gs^2_{DN} + dgs^2_N$
Days D	$d-1$	$\sum_{1}^{d}\left(\sum_{1}^{ng} x\right)^2/ng - CT$	$\frac{SS_D}{(d-1)}$	$s^2_e + ns^2_{DG} + gs^2_{DN} + ngs^2_D$
Operators times grains	$(n-1)(g-1)$	$\sum_{1}^{ng}\left(\sum_{1}^{d} x\right)^2/d - CT - SS_N - SS_G$	$\frac{SS_{NG}}{(n-1)(g-1)}$	$s^2_e + ds^2_{NG}$
Operators times days	$(n-1)(d-1)$	$\sum_{1}^{nd}\left(\sum_{1}^{g} x\right)^2/g - CT - SS_N - SS_D$	$\frac{SS_{ND}}{(n-1)(d-1)}$	$s^2_e + gs^2_{ND}$
Days times grains	$(d-1)(g-1)$	$\sum_{1}^{dg}\left(\sum_{1}^{n} x\right)^2/n - CT - SS_D - SS_G$	$\frac{SS_{DG}}{(d-1)(g-1)}$	$s^2_e + ns^2_{DG}$
Days times grains times Operators	$(n-1)(g-1)$ $(d-1)$	$\sum_{1}^{ndg} x^2 - CT - SS_N - SS_G - SS_D - SS_{ND} - SS_{NG} - SS_{DG}$	$\frac{SS_{NDG}}{(n-1)(g-1)(d-1)}$	s^2_e
TOTAL	$ngd-1$	$\sum_{1}^{ndg} x^2 - CT$		

Where $CT = \left(\sum_{1}^{ndg} x\right)^2/ndg$; n = no. of operators; g = no of grains; d = no of days

The separate sources of variation may now be arranged in tabular summary (table 1) as an analysis of variance table (Fisher, 1948) in which the design of experiment and the resulting statistical analysis is summarized. The following re-

marks are appended by way of brief explanation of the meaning of each source of variation; for greater details the interested reader may be referred to relevant statistical texts (Fisher, 1948; Snedecor, 1946; Youden, 1951, etc.) and, for examples of similar procedures applied to geological problems, to a growing number of publications (Swineford and Swineford, 1946; Chayes and Fairbairn, 1951; Miller, 1949; Griffiths and Rosenfeld, 1950; Rosenfeld and Griffiths, 1950).

The main effects arising from differences between grains, operators and days respectively may appear to carry fairly obvious implications, but the use of analysis of variance is best understood when it is realized that each main effect is used to test a separate null hypothesis (Fisher, 1949). Thus we may set up a null hypothesis concerning grains as follows:

> There are no differences between grains in sphericity or roundness; as a corollary the entire set of measurements is then a random sample of an infinite population of such measurements in which there are no differences in sphericity or roundness as the case may be.

Our test consists of an attempt to disprove this hypothesis. The setting up of a null hypothesis actually implies that we have defined a population of which our set of measurements is a random sample, and our test comprises computation of statistics, the mean and variance, from the sample and estimation of how likely such statistical values are to arise on the basis of our hypothesis about the population.

In the present case, as implied by the title "analysis of variance," we shall be testing the variance around the means and using the mean and variance but especially the latter as a basis of judgment of whether the sample of measurements has indeed been drawn from a population in which the differences in sphericity are non-existent. Each main effect leads to a separate null hypothesis and, of course, to a separate population for testing. The interactions are, however, somewhat more intricate; we may compare, by way of illustration, the main effect differences between operators and the first order interaction operators times grains. There may be and, indeed, usually are differences between operators in which one operator always estimates, say, a high value of sphericity whereas another measures a low one (see, for example, operators in figure 5). In addition some operators estimate consistently, i.e., always

yield high or always low values, whereas others are inconsistent, sometimes measuring high and at other times measuring low values. The differences between operators is estimated as the main effect whereas the inconsistency of operators from grain to grain is measured by the interaction term operators times grains. It is reasonably simple to extend this argument to variations of operators from day to day and so to account for operators times grains and operators times days first order interactions. There appears to be no obvious physical meaning to the first order interaction of grains times days principally because it is difficult to conceive of a grain varying from day to day or being inconsistent in this sense; nevertheless, this term is mathematically separately evaluated here although it is customary to include the variance from this source with "error" in the usual case. The reason for this departure from convention is twofold; firstly, it is simpler to explain the analysis in this way and, secondly, we are in such a dubious position geologically in terms of evaluating what is and what is not important that it behooves us to be extremely cautious in attempts to simplify by jumping to conclusions where we have no prior experience to guide us. Although, therefore, we cannot at present imagine a meaning for such a source of variation, it is as well to be aware of its possible eventuality in future experiments!

Finally, the second order interaction entitled grains times operators times days includes all variation from unassigned sources. This then becomes our basic error term for the experiment. It may be as well to add that this term is capable of physical interpretation in experiments possessing a more complex design and has been of value in evaluating certain agricultural experiments. In the present case, however, we do not attempt to interpret the physical meaning of the term but accept it as a measure of experimental error. The magnitude of this error determines the sensitivity of the entire experiment.

TREATMENT OF THE DATA

The use of analysis of variance technique carries with it certain implications concerning the variation in the measured variable (Eisenhart, 1947); ideally the frequency distribution of the measurements should follow the normal distribution but considerable departure from this ideal is permissible in practice.

Nevertheless, it is necessary to ensure that the data are suitable for this kind of statistical treatment, and certain standardized tests exist for this purpose. Furthermore, it is always advisable to be aware of the frequency distribution of the measured variable because this information is of considerable importance both in understanding the significance of the measurements and in evaluating the usefulness of the technique.

The scales of measurement of both sphericity and roundness extend from 0 to 1 and are, therefore, "percentage" or proportion type scales, whereas the limits of the ideal normal distribution extend from minus to plus infinity; as a result, the frequency distributions of sphericity and roundness may show cutoffs at zero and unity. The recommended practice is to transform the variable from percentage or proportion to angular values (the inverse or arc sin transformation), and tables to facilitate this transformation may be found in many standard texts (Fisher and Yates, 1948; Snedecor, 1946; etc.).

In using analysis of variance techniques a common error variance is computed as the basis for comparison of all main and subsidiary effects, and for this reason it is necessary to ensure that the variance and especially the error variance is not related to the mean; the independence of mean and variance in the normal distribution is, of course, a fundamental property of such a frequency distribution. In order to ensure as near fulfillment of these prerequisites as possible, it is necessary to examine the frequency distribution of the variable in some detail, and should any doubt concerning the requirements exist, it is advisable to transform the variable to a scale in which as many as possible of the assumptions involved in using analysis of variance technique are fulfilled.[2]

Sphericity measurements.—The 441 values of sphericity compiled for this investigation are listed in table 2 and are graphically displayed as a histogram in figure 1, together with the computed normal curve possessing the same mean and standard deviation. The importance of the departures may be estimated in several ways and here the chi-square test was

[2] A more detailed and formal discussion of this and other aspects of analysis of variance is included in a series of articles in Biometrics Bulletin (Eisenhart, 1947; Cochran, 1947; and Bartlett, 1947), and the use of the inverse sin transformation is more fully ventilated in "Selected techniques of statistical analysis," Eisenhart, C., et al., McGraw-Hill Book Co., 1947, part 3, p. 395-416.

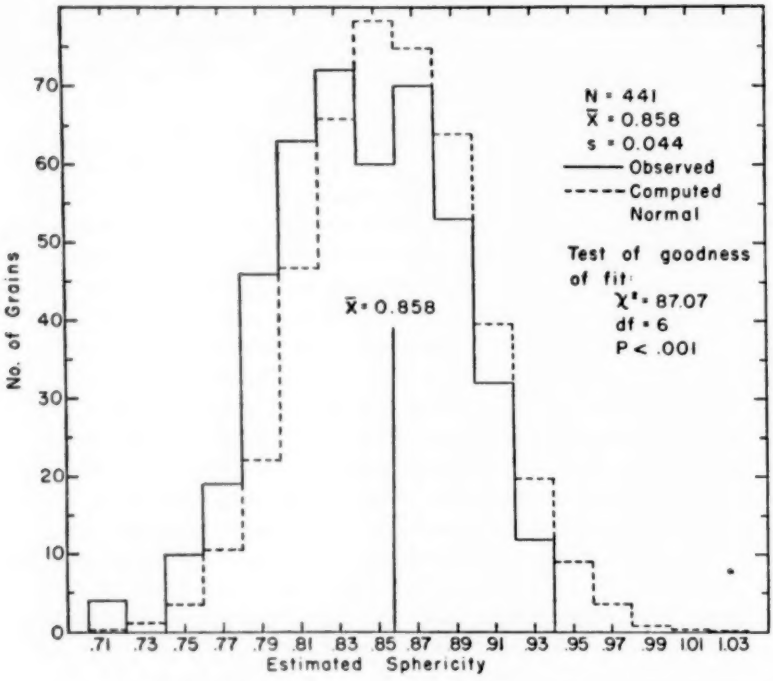

Fig. 1. Histogram of raw data sphericity values.

used (Dixon and Massey, 1951, p. 190). The computed chi-square value yields a probability of less than .001; this may be interpreted to signify that if our 441 values were a random sample from a normal population of the same mean and standard deviation, such a chi-square would occur less than once in every 1000 samples. We judge, therefore, that our 441 values depart significantly from a normal distribution.

If we now calculate the mean and variance for each of the 21 grains based on the 21 independent estimates for each grain, we find that the variance shows no obvious relationship to the mean. For our present purpose and based on this limited set of data we may deduce that, although the departure from normalcy is significant, the effect on the levels of significance in the analysis of variance and particularly the F test (Snedecor, 1946) is not serious. Nevertheless, for purposes of exploration, each value was transformed to its arc sin equivalent and

TABLE 2

Raw Data Sphericity Values (x100)

Operator	A			B			C			D			E			F			G		
Day	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Grain																					
1	91	91	89	81	87	79	87	90	89	79	87	83	83	89	85	87	91	87	89	87	87
2	81	81	83	79	81	81	83	83	81	83	85	87	85	79	83	83	87	85	85	85	85
3	81	81	81	75	77	77	79	77	79	83	83	85	81	79	81	83	83	85	79	83	81
4	87	91	85	71	87	83	87	83	87	83	85	85	83	91	83	87	87	87	87	85	87
5	77	75	79	85	77	75	75	77	77	79	81	79	71	79	75	83	79	81	71	79	77
6	87	89	89	87	85	87	93	87	91	83	89	91	81	89	89	91	89	91	87	87	89
7	85	83	85	81	85	81	83	79	85	91	85	85	79	87	81	87	81	87	87	85	87
8	89	87	89	83	81	81	85	85	87	87	89	91	83	85	81	89	91	89	83	87	87
9	83	81	83	77	83	77	79	77	79	83	79	83	81	80	77	81	79	77	81	79	83
10	85	83	85	79	85	81	75	81	83	81	83	87	83	84	75	89	87	87	85	85	81
11	87	85	87	77	81	83	83	81	85	81	81	81	85	83	79	89	87	87	79	83	85
12	85	83	81	81	85	79	77	77	85	83	83	87	83	81	81	83	79	83	79	83	81
13	81	83	83	79	79	79	79	80	81	77	81	85	87	78	79	81	79	83	75	81	79
14	85	87	85	83	85	83	83	81	79	87	71	87	81	81	75	85	89	89	79	81	81
15	83	87	85	79	79	79	85	85	85	83	81	89	77	82	81	81	83	87	81	79	79
16	89	91	89	87	87	87	91	89	93	91	87	93	87	89	90	93	89	93	89	91	91
17	89	87	89	87	87	87	89	89	89	91	87	89	91	90	91	89	87	87	91	89	89
18	89	91	89	85	91	89	85	85	89	93	87	91	89	89	89	91	91	89	87	89	87
19	87	85	85	79	81	81	81	83	83	83	85	83	83	83	77	81	85	83	83	87	85
20	85	85	85	75	81	79	83	85	87	83	83	83	79	81	79	87	87	83	83	83	83
21	93	93	91	81	89	85	89	89	91	89	93	93	87	87	89	93	93	91	89	89	89

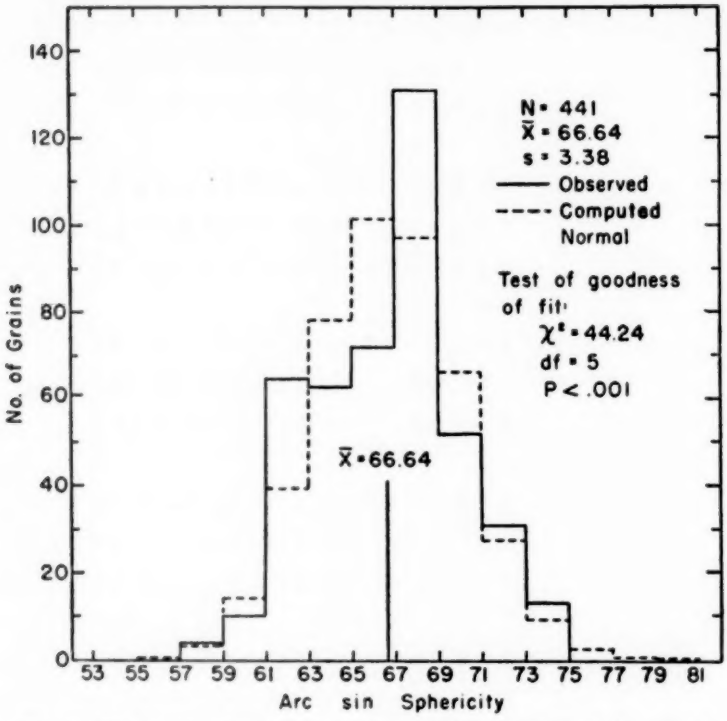

Fig. 2. Histogram of sphericity values—data transformed to arc sin.

the analysis of variance performed on both raw and transformed data. The comparison of the results follows later (table 5).

As a matter of interest the histogram of the transformed data is given in figure 2, together with the normal curve computed for the same mean and variance. The chi-square test yields a probability value of less than .001. It seems clear that the arc sin transformation in this case did not suffice to normalize the data. It should be noted from the frequency distribution of the raw data that the mean value of 0.858, the almost symmetrical histogram, and the lack of cutoffs at either end suggest that no transformation is necessary in this case.

Roundness measurements.—The roundness data based on the 441 estimates is given in table 3 and plotted as a histogram in figure 3. This distribution tested against the normal curve

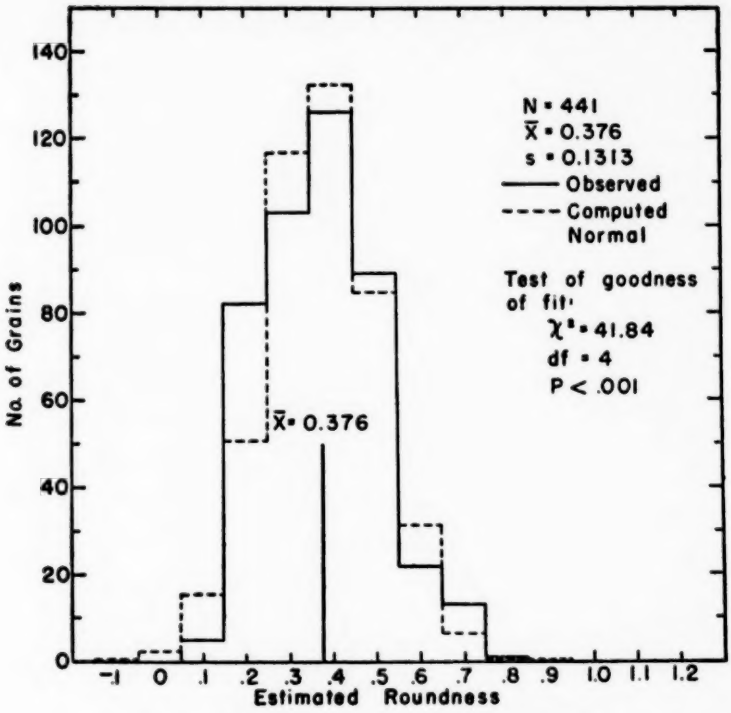

Fig. 3. Histogram of raw data roundness values.

computed for the same mean and variance yields a chi-square of 41.84 and P <0.001, again indicating a non-normal distribution. Furthermore, a plot of the mean roundness against variance for each of the 21 grains yields a definite trend (fig. 4). In this case the association is estimated by the correlation coefficient $r = -0.4272$ which, on the basis of 21 sets of values, yields a P of $<0.1> 0.05$ (Fisher, 1948, table 6, p. 209). Although not significant at the conventional level, such a degree of association is clear indication that the mean and variance are not independent in this set of data. Transformation to inverse sin equivalents failed either to improve the normalcy or to render the mean independent of the variance. It was decided, therefore, to use Bartlett's approach (1947, p. 39) and to calculate a correction from the data themselves. The trend of mean versus variance was assumed linear and com-

TABLE 3

Raw Data Roundness Values (x 10)

Operator	A			B			C			D			E			F			G		
Day	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Grain																					
1	3	4	3	2	2	2	4	5	5	2	4	2	2	2	2	4	5	3	2	2	2
2	2	3	3	3	3	4	4	4	4	4	5	4	3	3	2	2	5	3	3	3	2
3	5	4	5	4	5	5	5	5	5	4	5	5	5	4	4	5	6	5	4	4	4
4	2	3	3	1	3	2	2	3	2	3	3	4	2	2	2	3	2	2	2	3	1
5	3	2	3	3	2	2	4	5	4	2	3	3	1	2	2	2	3	3	2	5	2
6	3	5	2	2	3	3	3	3	4	4	4	5	2	2	2	3	2	2	3	2	2
7	4	4	4	4	4	4	5	3	5	5	5	5	4	3	3	5	4	3	3	4	4
8	2	5	2	2	4	3	3	2	2	3	4	5	2	2	2	3	2	2	2	2	3
9	4	3	2	2	3	3	3	3	3	4	4	4	2	2	2	4	3	3	2	1	2
10	4	4	5	3	5	3	5	5	5	5	5	4	2	4	3	5	4	5	2	3	4
11	5	5	5	3	3	4	3	4	4	3	4	4	3	4	4	5	5	5	3	3	4
12	6	6	6	7	8	7	6	6	7	6	7	7	6	5	6	7	7	7	5	5	5
13	4	4	4	4	3	4	4	4	4	5	3	4	4	2	3	4	4	4	4	4	3
14	2	4	3	3	2	2	3	3	3	4	2	2	2	2	2	2	2	3	1	3	2
15	3	4	4	4	4	3	5	2	4	2	3	4	2	3	4	5	4	4	3	4	3
16	3	4	3	5	4	3	3	3	4	5	3	5	4	4	3	2	2	3	4	2	3
17	6	6	6	6	6	5	7	7	7	7	6	7	5	6	5	6	5	6	6	6	6
18	4	5	4	5	4	4	4	5	4	5	5	5	4	4	3	2	3	4	3	4	4
19	5	4	4	3	5	4	4	4	4	4	4	5	4	4	4	3	3	3	4	2	4
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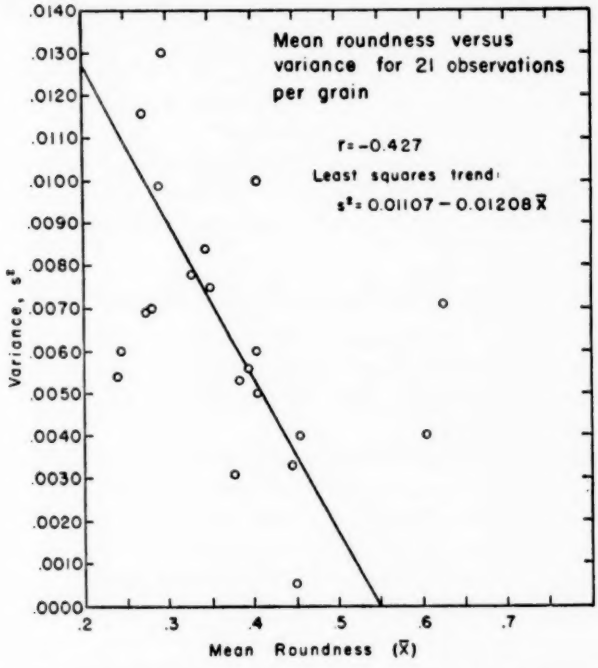

Fig. 4. Plot of mean roundness versus variance for 21 observations per grain.

puted by least squares (see fig. 4) yielding $s^2 = 0.01107 - 0.01208\bar{x}$, where s^2 = variance, and $\bar{x}$ = mean of the roundness values for each grain respectively. Following Bartlett's recommendations (op. cit. p. 39) the linear relationship was integrated to yield the required transformation:

$$P_A = \sqrt{0.01107 - 0.0121P} \qquad (1)$$

where P = measured roundness value and P_A = adjusted roundness value. Again the analysis of variance was computed on the basis of raw data, inverse sin and adjusted values as a basis for comparison (table 7). It should be noted that, using the computed formula given in equation (1) above, any roundness value greater than 0.7 will yield a negative adjusted value, i.e., this transformation is not suitable for adoption other than with the data herein recorded. Needless to say, the regression

adjusted roundness values show a mean independent of the variance.

In conclusion, it should perhaps be emphasized that the requirement of normalcy, while desirable, is not so essential as that the variance should be independent of the mean, and this is particularly true if, as in the present analysis, one is more interested in the estimation of components of variance than in the difference between means (see, in particular, Eisenhart, 1947, p. 19).

SOURCES OF VARIATION IN SPHERICITY

The analysis of variance.—The analysis of variance of the sphericity data in terms of the transformed (arc sin) values is summarized in table 4. The steps in the analysis may be outlined as follows:

From the components of variance (see mathematical model table 1) it can be seen that each source of variance may be tested against its appropriate error term; thus, the first order interactions are tested against the second order interaction. The actual test is performed by comparing the mean squares and computing the variance ratio or F value (Snedecor, 1946, p. 219). For example, if we divide the mean square for the days times grains interaction by the mean square for the second order interaction we are evaluating the variance ratio, $F=(s^2_e+7s^2_{DG})/s^2_e$; returning to our initial argument, if there are no differences, then these two independent estimates are estimates of the same variance. In the ideal case, s^2_{DG} is zero and the variance ratio becomes (s^2_e/s^2_e) which is, of course, unity. Departures from this ideal value which arise from chance causes or from random sampling of a homogeneous population are estimated by the appropriate F distribution. In the present case the F value of 1.04 is a near estimate of unity; in fact, the 20% point of the F distribution with 40 and 200 degrees of freedom is approximately 1.208,[3] and hence we would expect such an F value more than 20 times in every 100 of such tests in sampling from a homogeneous population. It would appear, therefore, that on the basis of the present evidence, variation from inconsistency of grains from day to day adds nothing more to the variation than that contained in the error

[3] Computed using Fisher's formula, Fisher and Yates, 1948, p. 34, footnote to table.

TABLE 4

Analysis of Variance

Sphericity of 21 Grains Determined by 7 Operators on each of 3 Days (arc sin transformation)

Source of Variation	DF	Sum of Squares	Mean Square	F	$F_{.05}$	$F_{.01}$	$F_{.001}$	DF used for test
Between Grains	20	3204.397	160.2198	27.137***	1.83	2.34	3.02	20/120
Between Operators	6	532.77	88.795	6.575**	3.00	4.82	8.38	6/12
				15.04***	2.17	2.96	4.04	6/120
Between Days	2	25.845	12.992	3.88*	2.99	4.60	6.91	2/12 2/280
Operators times Grains	120	708.509	5.904	1.7735***	1.35	1.53	...	120/280
Operators times Days	12	162.049	13.504	4.056***	1.83	2.34	3.02	12/280
Days times Grains	40	138.128	3.4532	1.04	1.45	1.69	...	40/200
Days times Grains times Operators	240	793.932	3.308					
Total	440	5565.63	12.6492					

CT = 1,967,941.36

Pooled Error Term = 3.329; DF 280

* = Significant at 5.0% level
** = Significant at 1.0% level
*** = Significant at 0.1% level

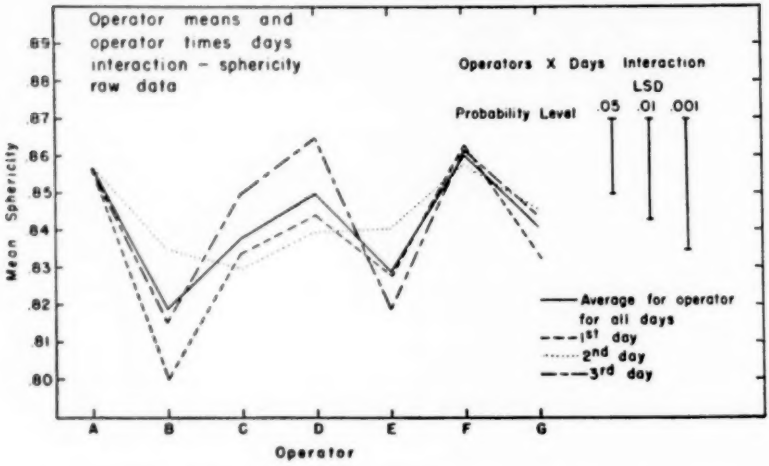

Fig. 5. Variation in sphericity for each operator, and for each operator on each day.

variance. In such a case we are justified in combining these two estimates to obtain a more efficient estimate of the error variance. The pooled estimate is found by adding the sums of squares for each term and dividing by the summed degrees of freedom; we obtain a mean square of 3.329 with 280 degrees of freedom and this value is recorded at the foot of table 4.

The next step is to test the interaction operators times days and this yields an F value of 4.056 when tested against the pooled error. The 0.1% point of the F distribution for 12 and 280 degrees of freedom respectively is 3.02, and hence we would expect such a large F value as that found very much less often than 1 time in 1000. We cannot but conclude that our initial hypothesis of no differences in the population is probably false in this case. Hence this interaction, the inconsistency of operators from day to day, contributes a significant amount of variance to the total. The variation of each operator for each day is illustrated in figure 5. Similarly the operators times grains inconsistency is significant at the 0.1% level.

In attempting to evaluate the main effects we have a choice of two tests and each test supplies somewhat different information. Thus in the case of differences in sphericity from day to day the logical error term might be days times grains or days

times operators. Let us assume for the moment that differences between operators are omitted from consideration (e.g., the tests were performed by a single operator); then we may test differences between day means against days times grains or the more efficient estimate of the pooled error. The resultant F value of 3.88 exceeds the 5% F value of the equivalent F distribution ($F_{05} = 2.99$, $F_{01} = 4.60$) but is less than the 1% value. It seems clear that the differences between days significantly exceeds the experimental error and this suggests that it may be as well to include in any subsequent experiment a more extended period to evaluate differences between days before concluding that this source of variation may be neglected.

The second test of differences between the means for days may be made by comparison against the inconsistency of operators from day to day, and this yields an F value of less than unity. In effect, then, the contribution to variation of the inconsistency of operators from day to day reduces the contribution due to differences between days to negligible proportions.

The conclusion to be drawn from these two tests is that with a number of operators the inconsistency of operators from day to day is sufficiently large to overshadow differences due to measurements performed on different days; whereas when operator difference is removed, such as occurs when the tests are performed by a single operator, the day to day differences are sufficiently large to necessitate separate evaluation. These findings need confirmation from a more extended test over a longer period before final acceptance.

Similarly, differences between operators may be tested both against operators times days and operators times grains interaction. In both cases the results are substantially similar, both F values being significant beyond the 1% value of the corresponding F distributions.[4] Differences between operators are clearly established. The variation in operator means is contained in the graph of figure 5 (solid line).

Differences between grains tested against operators times grains interaction are also highly significant, the resultant F value far exceeding the 0.1% value of the F distribution for 20 and 120 degrees of freedom. The differences between grain means are illustrated in figure 6.

[4] In fact, operator differences tested against operator x grains interaction is significant beyond the 0.1% level.

TABLE 5

Comparison of the Analysis of Raw and Transformed Data for Sphericity

Source of Variation	DF	Arc Sin Data F	Arc Sin Data Sig. Level	Raw Data F	Raw Data Sig. Level
Between Grains	20	27.137	$P < 0.001$	27.47	$P < 0.001$
Between Operators	6	6.575[a] 15.04[b]	$P < 0.01$ $P < 0.001$	9.25 16.90	$P < 0.001$ $P < 0.001$
Between Days	2	0.96[c] 3.90[d]	Not sig. $P < 0.05$	1.41 3.99	$P > .20$ Not sig. $P < 0.05$
Operators x Grains	120	1.774	$P < 0.001$	1.552	$P < 0.01$
Operators x Days	12	4.056	$P < 0.001$	2.836	$P < 0.01$
Days x Grains	40	1.04	$P > 0.20$ Not sig.	0.875	Not sig.
Error Stand. Dev.	..	0.024[e]	.. ...	1.82	.. ...
Coeff. Variation	..	2.87[e]	.. ...	2.73	.. ...

a — tested against ops x days 6/12 DF

b — tested against ops x grains 6/120 DF

c — tested against ops x days 2/12 DF

d — tested against pooled error term 2/250 DF

e — the computations leading to analysis of variance were performed on coded sphericity values where $\Psi^1 = (\Psi_{\%} - .70)100$ whereas the error standard deviation and coefficient of variation have been decoded.

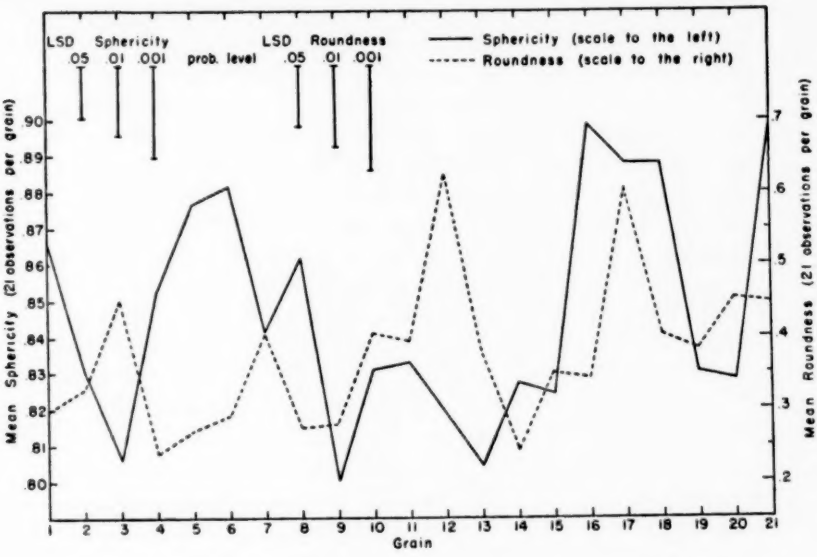

Fig. 6. Variation in mean sphericity and roundness among grains.

Comparison of the analysis of raw and transformed data.—The analysis of variance performed on the raw data and on the data transformed to its arc sin equivalent is displayed in table 5 in terms of the resulting F values and their corresponding significance levels. It can be seen that only minor changes in levels of significance occur and no differences would arise in the conclusions drawn from the analysis of the data based on either set of values. This is only to be expected in this case where the original frequency distribution does not suffer from the kind of aberration which the inverse sin transformation is used to ameliorate. Nevertheless, it is comforting to know that no great change occurs in using the transformation where it is not essential; if, for example, two sets of data were to be compared, one of which necessitated the arc sin transformation and the other did not, such a property of the transformation is a distinct advantage.

Estimation of components of variance in sphericity.—One of the advantages of the use of analysis of variance in the treatment of data arises from the fact that a suitable estimate of the variance contributed by each source may be obtained

from the data. The items contributing to the variance of each factor together with their weighting coefficients are listed opposite the respective sources in the mathematical model of table 1. It will be noted, for example, that the variance arising from variation of operators from day to day—operators times days interaction—is composed of a contribution from error (s^2_e) and a second contribution from the interacting items (s^2_{ND}). The second component in this variance is based on the 21 grain means and the coefficient g is used to make the estimate conform with the error variance.[5] Thus we have an estimate of the operators times days interaction as $s^2_e + gs^2_{ND}$; we also have an estimate s^2_e for the error variance, and so the magnitude of the variance for the interaction (s^2_{ND}) may be obtained by simple arithmetic. The variance for each term in the analysis is listed in table 6.

It can be seen that the largest contribution to variance, or the greatest source of variation, arises from differences between grains, and this is almost twice the next largest contribution from unassigned sources—the error term. The third largest contribution is from differences between operators and this is about one-fifth of the variance from differences between grains.

These components of variance clearly indicate that there must be many sources contributing to error, and from the magnitude of this residual variation there are other effects which could be profitably introduced into the analysis. In addition, this analysis of the variance components suggests that should the range of variation in sphericity between grains be reduced then the differences between operators will show a relative increase.

Again it can be seen that the variance arising from inconsistency of operators from grain to grain is about twice as large as that from the inconsistency of operator from day to day. The variance contributed by differences between days is negligible.

[5] This rule arises from the relationship between the variance of means and the variance of the raw items, i.e., $ns^2_{\bar{x}} = s^2_x$ where $s^2_{\bar{x}}$ is the variance between the means, s^2_x is the variance of the original items, and n is the number of items used in calculating each mean (Crump, 1946).

TABLE 6

Components of Variance in Sphericity Estimation

Source of Variation	Coded Raw Data			Decoded	Inverse Sin Data		
	Significance Level P	s^2	s	s	Significance Level P	s^2	s
Between Grains	< 0.001	11.2590	3.35	0.033	< 0.001	7.3415	2.71
Between Operators	< 0.001	2.0877	1.44	0.014	< 0.001	1.1542	1.07
Between Days	N. S.	0.0455	0.21	0.002	N. S.	0	0
Operators x Grains	< 0.01	1.0234	1.0117	0.0101	< 0.001	0.8583	0.93
Operators x Days	< 0.01	0.4981	0.7058	0.007	< 0.001	0.4845	0.6961
Grains x Days	N. S.	0	0	0	N. S.	0.0207	0.1440
Error	....	5.8601	2.4208	0.024	....	3.3080	1.8188
		Coded		Decoded			
$\overline{X}$	....	14.2290	.....	0.842	....	66.8016	.....
Cv %	....	.....	.....	2.87	....	2.73	.....

The standard deviation derived from the variance is also listed in original units in the column headed "decoded s" but it must be remembered that the relationships between the standard deviations are not as simple as those between the variances. The coefficient of variation is computed from the error standard deviation (square root of the error mean square) and the grand mean of the entire experiment[6] and is considered to be a reflection of the experimental control; in general, a coefficient of variation less than 20% is reasonable and less than 10%, good.

The comparison of variance components based on estimation using the raw data and the transformed data are also included in table 6. The relative magnitudes are certainly not the same, but the conclusions based upon them are, so that, in this case, the use of the transformation is evidently not necessary.

It must be emphasized that this simple technique for the evaluation of the variance contributed by each factor in the analysis is based upon assumptions concerning the contributions to variance in the original data. One such assumption, implicit in the manipulations leading to separate evaluation of the variances associated with each factor, is the additivity of the variance from each source. If the original frequency distribution were normal such conditions would hold, but in non-normal distributions it is necessary to demonstrate the independence of mean and variance for such conditions to hold. Transformation of the variable is one approach to this problem and accounts for the attempt to use the inverse sin conversion in this investigation (Bartlett, 1947; Eisenhart, et al., 1947). In the case of sphericity determination, however, although the frequency distribution is non-normal, the mean does not appear to be related to the variance in any systematic manner. The use of the arc sin transformation is not therefore mandatory in this case. In general, however, it is as well to ascertain empirically that additivity is likely before the analysis of components of variance is attempted (Crump, 1946).

[6] According to the formula $Cv\% = \frac{\text{E.M.S.}}{\overline{X}_G} \times 100$; where Cv = coefficient of variation, E.M.S. = error mean square, and $\overline{X}_G$ is the grand mean.

TABLE 7

Analysis of Variance

Roundness of 21 Quartz Grains Determined by 7 Operators on each of 3 Days (regression adjusted data)

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Squares x 10^4	F	$F_{.05}$	$F_{.01}$	$F_{.001}$	DF used for test
Between Grains	20	0.029828	14.914	28.96***	...	...	2.40	24/120
Between Operators	6	0.004612	7.686	14.92***	...	...	4.04	6/120
Between Days	2	0.000049	0.245	1.0	...	...	...	2/120
Grains times Operators	120	0.006180	0.515	1.9360***	1.32	1.48	...	120/240
Operators times Days	12	0.000401	0.334	1.255	1.80	...	...	12/240
Days times Grains	40	0.000910	0.2275	0.8553	...	...	...	40/240
Days times Grains times Operators (Error)	240	0.006373	0.2655	.....	...	...	...	.....
Total	440	0.048353	1.0989					

Pooled Error Term = 0.2631; DF 292

SOURCES OF VARIATION IN THE VISUAL ESTIMATION OF ROUNDNESS

The analysis of variance.—The analysis of variance of roundness in terms of the regression adjusted data is illustrated in table 7 and it will be unnecessary to repeat the steps in detail because the same procedure is followed as that described in the analysis of the sphericity data. The results of the tests and minor modifications appropriate to the different data are outlined in the following paragraphs.

Testing the days times grains and operators times grains mean square against that for error yield F values which are not significant. The variance for these terms may then be pooled, and the pooled error is recorded at the foot of table 9. Little is gained by this manipulation, and for subsequent tests the unmodified error term is used for testing the grains times operators interaction. This is significant beyond the 0.1% point of the F distribution for 120 and 240 degrees of freedom, and the F value would only be increased by using the pooled error term. Clearly the inconsistency of operators from grain to grain makes a considerable contribution and becomes the basic error for comparison with main effects.

The difference between days yields an F value less than unity when tested against either the pooled error or the grain times operator interaction. Differences from day to day are therefore not significant.

Differences between operators and between grains are each significant beyond the 0.1% level of the respective F distributions. Hence these differences are highly important; the magnitudes of variation for these two main effects are included in figures 7 and 6 respectively.

Comparison of the analysis of raw and transformed data.—The comparison of the results of analysis of variance using raw data, arc sin and regression adjusted values is summarized in table 8. No difference in the conclusions to be drawn from the analysis would arise whichever kind of data were followed. This serves to illustrate the degree of relaxation of the assumptions of normality which may be permitted without affecting the levels of significance in using the analysis of variance procedure for testing the differences between means.

Estimation of the components of variance in roundness.—An estimate of the contribution to variation arising from each term in the analysis is recorded in table 9, and the results

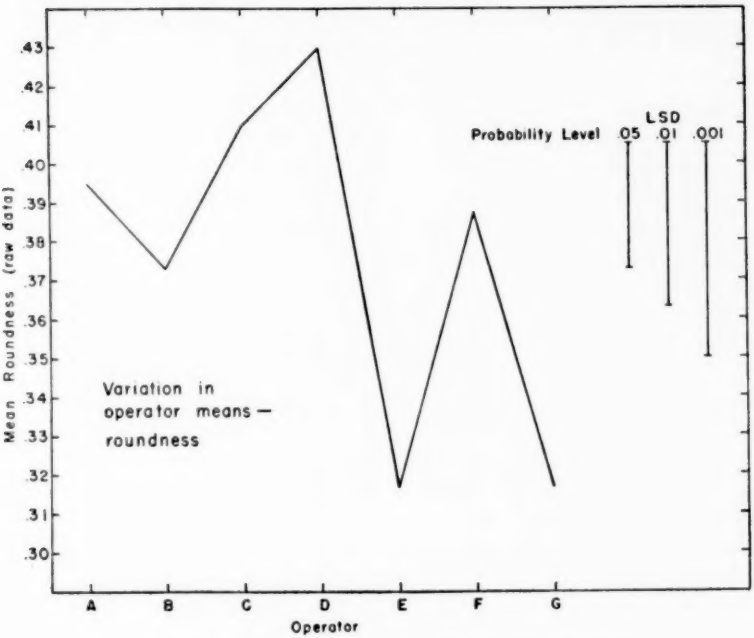

Fig. 7. Variation in mean roundness among operators.

of this computation using the raw data, the arc sin transformed data and the regression adjusted data are included for comparison.

The qualitative treatment of these values by comparing their relative magnitudes in terms of each of the respective scales would hardly differ in any important respect, but it should be emphasized that for further manipulations the most reliable estimate is that based on the regression adjusted data. This conclusion is predicated upon the fact that in both raw and arc sin data the mean is related to the variance and conditions upon which the computations of the estimates of variance are based do not hold. In the case of the regression adjusted data the mean is independent of the variance and additive relations between the components of variance are most likely. The subsequent discussion utilizes all three estimates, but attempts to compute the number of items necessary for the reduction of any variance component for future experiments must rely solely on the regression adjusted data.

TABLE 8

Comparison of Analysis of Variance of Roundness Estimation in Terms of Observed, Arc Sin and Regression Data

Source of Variation	DF	Observed Data F	Observed Data Significance Level P	Arc Sin Data F	Arc Sin Data Significance Level P	Regression Adjusted Data F	Regression Adjusted Data Significance Level P
Between Grains	20	27.77	‹0.001	13.33	‹0.001	28.96	‹0.001
Between Operators	6	14.509	‹0.001	27.01	‹0.001	14.92	‹0.001
Between Days	2	1.257	N. S.	1.380	N.S.	‹1	N. S.
Operators x Grains	120	2.142	‹0.001	1.714	‹0.001	1.936	‹0.001
Operators x Days	12	1.168	N. S.	0.827	N. S.	1.255	N. S.
Days x Grains	40	1.048	N. S.	0.876	N. S.	0.855	N. S.
Error	240						

The main source of variation arises from the differences in roundness between the different grains, and this term is at least twice that of the next largest source of variation, the error term. The variation associated with the error term is unassigned so that we may consider that the experiment has reduced the error variation to less than half of the variation we wish to determine. Evidently differences in roundness between grains may be established by the visual comparison technique.

The third source of variation in order of magnitude arises from differences between operators, and this is approximately one-sixth of the variation arising from differences in roundness between grains. The next source, the interaction term, operators times grains, also contributes an important part of the total variation. In this experiment the basic error term for the comparison of main effects is this interaction term, and hence the inconsistency of operator from grain to grain determines the level of sensitivity of the visual comparison technique. It can be realized from the form of the analysis that differences between operators can be segregated but their inconsistency determines the magnitude of the difference in roundness between grains which can be detected by this technique. It should be emphasized that any reduction in the variation in roundness between grains will result in a relative increase in magnitude of the inconsistency; hence in any cooperative experiment this term should always be included and evaluated as a measure of sensitivity of the experiment.

The other sources, differences between days, and the respective interactions, operators times days, and days times grains do not contribute in any appreciable degree to the variation and may be included in the error term without inflating that item if future experiments confirm the present results.

The recording of the decoded standard deviation in table 9 is included merely as an appropriate guide to the relative importance of the several factors. In these experiments as shown by the coefficients of variation the use of the regression adjusted data has resulted in an improvement in the experimental control.

DISCUSSION AND CONCLUSIONS

It should be clear from the preceding analysis that valid differences in both sphericity and roundness of quartz grains may be established by the use of visual comparison techniques.

TABLE 9

Comparison of Components of Variance in Roundness Estimation

Source of Variation	Raw data Mean square (x10⁴)	Raw data decoded s	Arc Sin Mean square	Regression adjusted Mean square (x10⁴)
Between Grains	105.73	0.103	38.633	0.686
Between Operators	17.69	0.042	6.138	0.113
Between Days	0.015	0.001	....	....
Operators x Grains	14.75	0.038	4.318	0.083
Operators x Days	0.310	0.006	....	0.003
Days x Grains	0.270	0.005	....	....
Error	38.75	0.062	18.152	0.265
Cv %	16.6	....	11.35	6.41

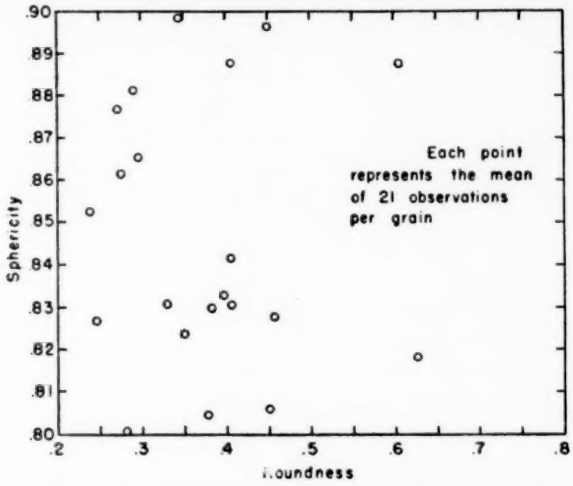

Fig. 8. Plot of mean sphericity versus mean roundness for 21 observations per grain.

It is, furthermore, confirmed that variation in roundness is larger than that in sphericity.

The magnitudes of the variation may be compared by means of figure 6, and the statistic "least significant difference" (LSD) may be considered as a very approximate measure of the least differences which can be detected in each case.[7] It would appear to be prudent to measure sphericity rather than estimate it, both because measuring techniques available are simple and rapid and because the entire range of variation in sphericity is not likely to be much larger than that included in this experiment.

In any future experiments some measure of operator variation should be included, and in cooperative experiments including more than one operator the inconsistency of operators from grain to grain should also be evaluated. It must be emphasized that where this has not been carried out the differences in either sphericity or roundness may not be solely ascribable to geological factors but include experimental or technique error.

[7] This measure should not be taken too literally because it achieves precise meaning only under very restricted conditions and particularly where the frequency distributions of the measured variable are homogeneous and closely approximate normality.

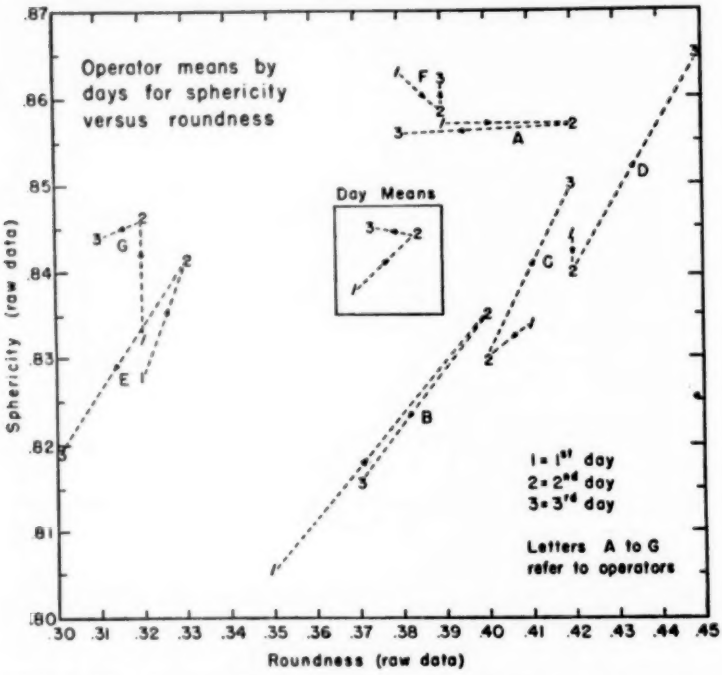

Fig. 9. Plot of mean sphericity versus mean roundness for 21 observations per operator.

It is often considered that sphericity and roundness tend to vary sympathetically. The means of the 21 independent estimates of sphericity and roundness for each of the 21 grains are plotted in figure 8. No obvious trend appears.

As a matter of interest, if we plot the mean sphericity and roundness for each operator, however (fig. 9), a distinct trend is shown. Firstly, the approximate slope of the trends suggests that operators tend to react in sympathy to visual estimates of roundness and sphericity. In other words, when an operator estimates high sphericity, he also tends to estimate high roundness and vice versa.

Secondly, there is a reversal in the case of five of the seven operators which shows that on the first and third occasions (days 1 and 3) three of the operators measured high sphericity and roundness, whereas on the second day these three estimated

relatively lower roundness and sphericity on the same grains. Two other operators showed the reverse tendency. This is presumably connected with an arrangement of the technique; some of the operators remarked that there was a tendency to compare the first grain with the comparison chart and thereafter to compare each grain with the preceding one. To counteract this effect it was suggested (by M.A.R.) that the operators run through the grains from numbers 1 to 21 the first day, reverse the order of grains the second, and return to the original order the third. The reversal of the estimates is doubtless connected with this reversal in arrangement. The inconsistency in the reaction of the three operators contrasted with that of the two operators reduced the effect of differences due to days—or more correctly, due to order of the grains.

As was noted during the analysis, the differences between day means in sphericity were significant at the 5% level (table 4), but the differences between day means for roundness were not significant. Similarly, variation of grains from day to day as estimated by the interaction term grains times days was not significant; nevertheless, plotting the day means (inset, fig. 9) shows again the sympathetic variation of sphericity with roundness.

The moral is obvious, a more extended test of day to day or order of treatment of grains is necessary before these items can be considered relatively unimportant. In practice it would be advisable to randomize the order of treatment of grains on each occasion. Furthermore, the tendency among operators to record high sphericity along with high roundness and vice versa when using visual comparison techniques suggests some care in accepting sympathetic variation in sphericity and roundness as an attribute of grains.

An interesting item in the estimation of roundness is emphasized by the relationship between the mean and variance of the 21 grains (fig. 4). The regression indicates that the variance increases with decreasing mean roundness, which in turn implies that operators can estimate with less ambiguity differences in roundness in well-rounded grains and are increasingly inconsistent as the grain becomes more angular. In other words smaller differences can be detected in grains of high roundness than in grains of low roundness. This is in complete contradiction to the conclusion stated by Pettijohn (1949, p.

51), " . . . because the eye can readily distinguish slight differences in roundness when the roundness values are low, but cannot make such distinctions when the values are large." If future experiments confirm our findings, then the roundness scale suggested by Pettijohn should be reversed and more classes included in the higher range of roundness than the lower. Theoretical analysis of these problems is always insidiously deceptive, but it does appear reasonable to conclude that small departures from a smooth rounded surface are easier to detect than on a serrated one! It seems abundantly clear to the writers that subjective classification, particularly by visual estimates, even with comparison scales such as the charts used in this experiment, are no guarantee of valid conclusions from the observations unless the experiment is suitably designed; it is essential to evaluate the different sources of variation and establish that the differences observed are associated with real differences in the variable estimated and not introduced by lack of control of the psychological bias present in every observer.

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FURTHER DATA ON THE SYSTEM Na_2O—CaO—SiO_2

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ABSTRACT. Primary phase fields, liquidus surface, and quintuple points have been determined in part of the system Na_2O—CaO—SiO_2 on the silica-poor side of the join $CaSiO_3$-Na_2SiO_3. A new ternary compound, probably $Na_4Ca_3Si_6O_{17}$, has been described. The solid solution limit of Na_2O in dicalcium silicate has been shown to approach 3 per cent. The significance of these results in relation to Portland cement clinker components has been discussed, and the conclusion reached that most of the Na_2O in Portland cement is contained in the dicalcium silicate phase.

INTRODUCTION

THE high silica portion of the ternary system soda—lime—silica is of fundamental importance to the glass technologist, containing as it does, the area of compositions of common glasses. This portion of the system has, therefore, attracted much attention. The initial work was that of Morey and Bowen (1925), who have thoroughly investigated the phase equilibria of the NS—CS—S[1] portion of the system. In this they found ten primary phase fields, including those of cristobalite, tridymite, quartz, αCS, βCS, NS, NS_2, and those of the three ternary compounds NC_2S_3, N_2CS_3 and NC_3S_6, the latter two of which had incongruent melting points. A further ternary compound, NCS, has been described fully, together with the other ternary compounds, by Wyckoff and Morey (1926). Thilo (1951) mentioned the compound NC_2S_2.

The aim of the present work was to extend the knowledge of the phase equilibria of this system into the area containing less than 50 per cent silica. In the high-lime portion of the triangle occur the fields of the cement compounds C_2S and C_3S. The side C-CS is part of the well known lime—silica binary system (Rankin and Wright, 1915; Greig, 1927) from which limits were obtained on this boundary of the fields for lime, C_2S, C_3S_2, and αCS. The join NS-CS forms part of the system investigated by Morey and Bowen, and gave limits for the αCS, NC_2S_3 and N_2CS_3 fields. The more detailed work done

[1] Throughout the paper, formulae will be abbreviated in this manner, viz: N = Na_2O, C = CaO, S = SiO_2.

here has led to their boundary between the N_2CS_3 and NS fields being slightly altered. This, however, has not affected the essential equilibrium relationships.

EXPERIMENTAL

For the preparation of the mixtures, sodium and calcium carbonates of analytical reagent quality were used. The source of silica was quartz crushed to pass through a 200 mesh sieve, washed with hydrochloric acid and then with distilled water. A typical analysis of this product was SiO_2 99.86%, R_2O_3 0.04%, CaO 0.03%, H_2O 0.05%. The three constituents were weighed in the required proportions, fused in a platinum crucible, and, when possible, quenched to a glass. Each mixture was crushed and fused four times to ensure homogeneity. All glasses in this part of the system crystallized and reached equilibrium quite readily below their liquidus temperatures. The glassy charges could therefore be used directly for the quenching experiments without the need for preliminary treatment to render them crystalline.

The primary phase fields, boundary lines, quintuple points, and liquidus temperatures were determined by the quenching method. Small charges, wrapped in platinum foil of thickness 0.0005 inch, were rapidly quenched in water after holding at a constant temperature until equilibrium was reached. The resultant material, commonly a glass plus one crystalline phase, was examined with the petrological microscope.

The electric furnaces were wound with platinum—20% rhodium alloy wire, and the temperatures regulated to within 1°C. by a bridge type electronic controller; the furnace winding formed one arm of the bridge. The power into the furnace was controlled by a continuously working reversible choke instead of the more usual method of shunting the heating current through an extra resistance. Temperatures were measured with platinum-platinum 13% rhodium thermocouples checked against calibrated standards.

Difficulty was experienced through loss of soda by volatilization from compositions of high melting points rich in this constituent. In the earlier part of the work mixtures containing over 10 per cent soda were analyzed after preparation, so that the starting compositions were accurately known. In the later stages mixtures were prepared at temperatures between 1100°C.

and 1150°C., when the vapor pressure of the Na_2O was too low to cause any appreciable loss. The losses during quenching experiments were not easily determined. Many quenches, however, were carried out in small buckets containing sufficient material for a soda determination. The only quenches seriously affected were in those parts of the NCS, NC_2S_2 and C_2S fields outside the 1300°C. isotherm and containing over 15 per cent Na_2O. For unanalyzed quenches in this region, allowance was made for the final composition by assuming losses similar to those which were determined. Throughout the investigation, heating times were kept as short as equilibrium requirements would allow.

RESULTS

In the part of the system investigated nine primary phase fields were encountered. These were of the compounds pseudowollastonite (αCS), dicalcium silicate (C_2S), C_3S_2, NS, NC_2S_3, N_2CS_3, NCS, NC_2S_2 and $N_4C_3S_8$. As the first seven of these compounds have been fully described in the literature, only observations pertinent to the present work will be recorded for them.

C_2S: In the higher temperature quenches, charges poor in soda gave rise to small individuals, almost spherical in shape, which became oval or slightly elongated in melts of high soda concentration. In all cases the crystals appeared to be the stabilized α-form.

αCS: Large, well-formed individuals of high birefringence. Hexagonal outlines were frequently seen.

C_3S_2: Fairly well-formed prismatic crystals of moderate refractive index and low birefringence.

NC_2S_3: Well-formed pseudo-cubic crystals of square outline and low birefringence.

N_2CS_3: Small isotropic octahedra.

NCS: Oval-shaped isotropic crystals.

NS: Needles or lath-like crystals of moderate birefringence.

NC_2S_2: This compound has not previously been described, although it is mentioned by Thilo (1951) in connection with hydrated soda-lime silicates. In these studies it appeared characteristically as lath-shaped crystals of low birefringence. Its refractive indices are close to 1.665, birefringence 0.003, and optical character biaxial with high optic axial angle. The extinction of the lathes is inclined, and the crystals are probably

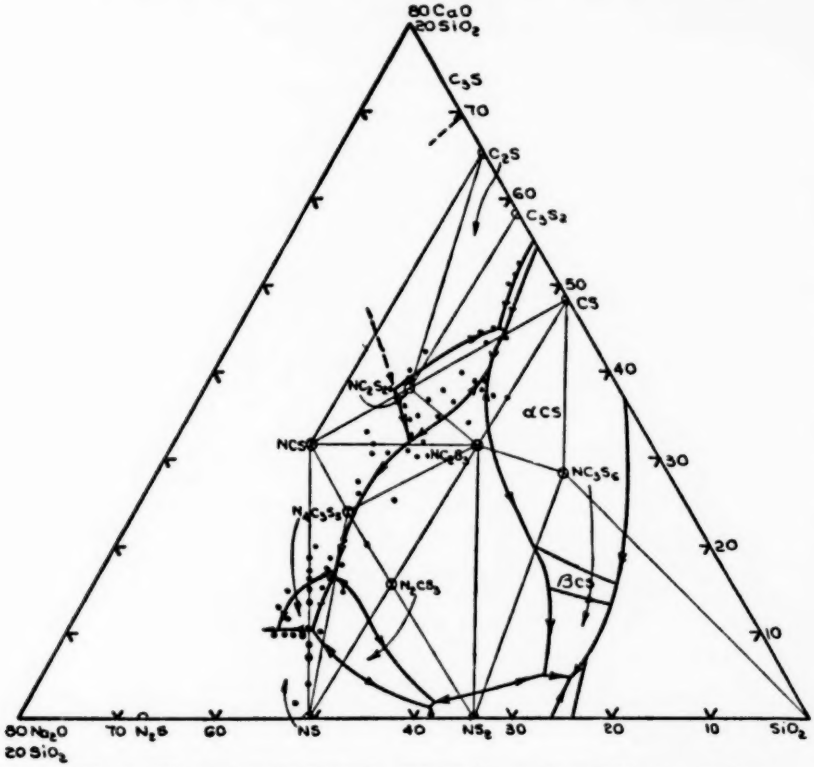

Fig. 1. The system Na_2O—CaO—SiO_2. Primary phase fields, composition triangles, and compositions studied. (The area NS-CS-SiO_2 after Morey and Bowen, 1925.)

monoclinic. Many crystals were twinned on a multiple law in a similar manner to plagioclase.

$N_4C_3S_5$: This previously undescribed compound was first seen in melts of high soda concentration as small rectangular crystals of very low birefringence and high refractive index. Near the liquidus temperatures of melts in which it occurred as a primary phase it formed large idiomorphic flat crystals showing what appeared to be two large basal planes with two sets of dome faces joining them. Its refractive indices are very close to 1.620, birefringence 0.002, biaxial +ve with high optic axial angle. The spacings (in Å) of lines observed in powder photographs of the last two compounds are given in appendix 2.

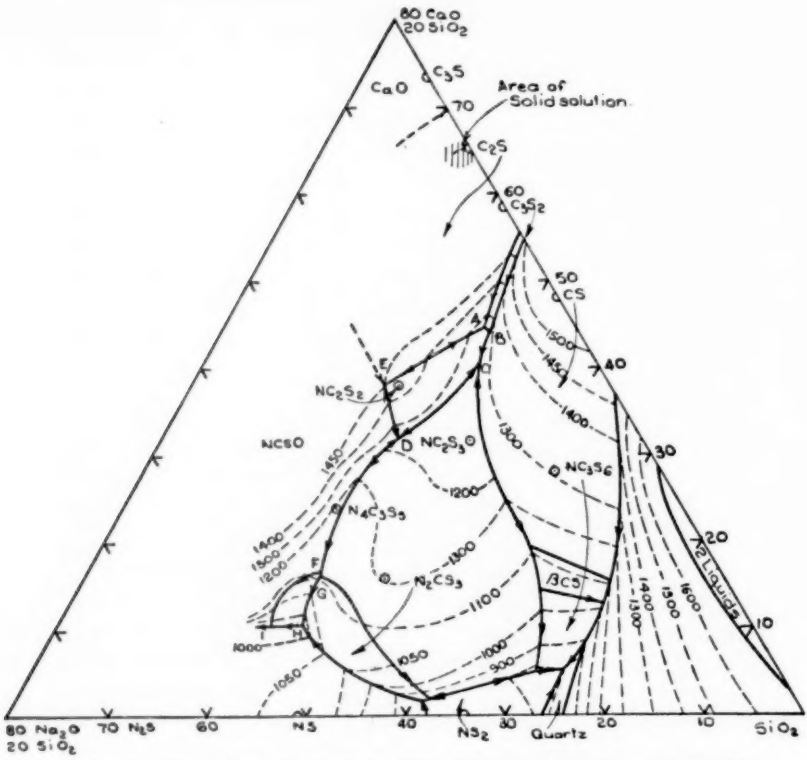

Fig. 2. The system Na_2O—CaO—SiO_2. Primary phase fields and liquidus isotherms. (The area NS-CS-SiO_2 after Morey and Bowen, 1925.)

The results of the more important quenching experiments are set out in appendix 1. The phase diagrams showing liquidus isotherms, composition triangles, phase boundaries and compositions studied are illustrated in figures 1 and 2. These diagrams extend the work of Morey and Bowen, which is embraced by the triangle NS-CS-S, into areas richer in soda and lime.

DISCUSSION

Dicalcium silicate.—Dicalcium silicate (Ca_2SiO_4) has attracted a great deal of attention due to its presence in a variety of industrial materials (e.g., cements, slags, refractories), and because of its complex solid phase changes (Bredig, 1943, 1950). The importance of this compound in the present system

lies mainly in its capacity to form solid solutions with small amounts of soda, and hence its relationship to the dicalcium silicate in high-alkali Portland cement clinkers. This solid solution has been recorded earlier by Greene (1944) and Newman and Wells (1946). The former prepared samples containing Fe_2O_3 or Al_2O_3 in addition to soda, and a dicalcium silicate containing 4.2 per cent Na_2O and 4.6 per cent Fe_2O_3 was identified. The latter authors state that "the limit of solubility of Na_2O in α-2 $CaO \cdot SiO_2$ is uncertain, but probably exceeds 1.3 per cent."

By measuring refractive indices of rapidly cooled preparations close to C_2S in the soda—lime–silica system, further data have now been obtained on the extent of these solid solutions. The refractive indices and birefringence of these samples are recorded in table 1. It was thought that the C_2S was in the α (hexagonal) form. The samples were prepared at 1400°C. and cooled rapidly. This temperature was probably above the $\alpha \rightarrow \alpha'$ inversion temperature for the solid solution, and neither exsolution effects nor twinning which might indicate inversion were seen.

The measurements in table 1 show that the birefringence and refractive indices decrease as the soda content increases at least up to 2.7 per cent. The maximum possible soda content

TABLE 1

Refractive Indices and Birefringence of Dicalcium Silicate Solid Solutions

Composition			Refractive Index		Birefringence
Na_2O	CaO	SiO_2			
* ..	65.1	34.9	1.716	1.736	0.20
..	62.0	38.0	1.703	1.731	.028
..	64.0	36.0	1.698	1.727	.029
..	66.0	34.0	1.712	1.734	.022
0.5	62.4	37.1	1.712	1.733	.021
0.8	63.2	36.0	1.709	1.729	.020
1.5	66.4	32.1	1.708	1.729	.021
1.7	63.3	35.0	1.705	1.720	.015
2.1	63.6	34.3	1.712	1.731	.019
2.1	62.0	35.9	1.709	1.728	.019
2.2	65.5	32.3	1.710	1.725	.015
2.7	62.3	35.0	1.698	1.713	.015

* dicalcium silicate composition.

is probably in the region of 3 per cent, and from the limited data available, seems to be greatest in the directions of the compounds NCS and NC_2S_2. The change in birefringence was well marked in the quenching experiments, where a distinct lowering of the interference colors of crystals was seen in primary phase C_2S quenched from melts containing quite small quantities of soda.

It seemed probable that at least some, and possibly a considerable part of the soda in Portland cement occurs in the dicalcium silicate. Ordinary Portland cement clinker generally contains about 25 per cent C_2S, so that ideally some 0.75 per cent Na_2O could be taken up in this compound. Any soda above this upper limit would have to be accounted for in other phases, such as the glassy interstitial material. The dicalcium silicate in three laboratory-prepared Portland cement clinkers of varying soda content was examined optically. The refractive indices are recorded in table 2, from which it is evident that the refractive index decreases with the soda content of the clinker. The only other constituents of the cement which might enter into the C_2S structure and affect its properties are Al_2O_3 and Fe_2O_3. Of these, the former would have little effect (unpublished work by the author on other materials containing Al_2O_3), and the latter would tend to increase the refractive index. The decrease in refractive index shown in table 2 can therefore be assigned with reasonable certainty to the effect of soda.

The optical properties tabulated in table 1 also show that the refractive indices are decreased and birefringence increased in soda-free compositions which contain an excess of silica over the stoichiometric composition. This may be due to a small

TABLE 2

Refractive Indices of Dicalcium Silicate from Laboratory Portland Cement Clinker

Clinker Number	Na_2O Content of Clinker	Refractive Indices of C_2S from Clinker	
26 B	2.4	....	1.71
27 B	1.0	....	1.72
22 B	$<$ 0.1	1.715	1.73

amount of solid solution with αCS, which has a birefringence of .044 compared with .020 for C_2S. Newman and Wells showed that the solid solution limit of SiO_2 in dicalcium silicate is 0.5 per cent (approximately 1 per cent CS). It may be noted that excess CaO or SiO_2 in the C_2S inhibited to a large extent the inversion to the γ form.

Equilibrium relationships.—These are summarized in figures 1 and 2 which show the primary phase fields, liquidus isotherms and composition triangles. Thirteen new boundary lines have been located in part or in full, and nine quintuple points. Of the latter, four were eutectics. Parts of nine primary phase fields have been mapped. Crystallization paths and other data may be interpreted from figures 1 and 2. A few points, however, deserve further mention.

The melting point of the compound NC_2S_2 ($Na_2Ca_2Si_2O_7$) was not determined with great accuracy owing to the high vapor pressure of soda at these temperatures. A sample of the compound quenched from 1446°C. was almost entirely glass, so that, allowing for a small loss of soda during heating, the melting point of the pure compound would be a little higher than this, and close to 1450°C. Its primary phase field is bordered by those of αCS, NC_2S_3, C_2S, C_3S_2 and NCS and four of the invariant points (B, C, D and E) at the corners of its field are eutectics.

The compound $N_4C_3S_5$ ($Na_8Ca_3Si_5O_{17}$) melts incongruently at 1125°C. It has a small primary phase field FGHI some distance from its composition point, which was located only with some difficulty. As there were no maxima on any of the boundaries of its field, it therefore probably lay to the right of the lines NS-F and F-NCS. Several possible compounds in this area were prepared by repeated sintering at 1000°C. and grinding, and it was found that the above composition satisfied the requirements of the compound. The product was composed almost entirely of the crystals obtained as a primary phase in the area GHI together with a small amount of glass of refractive index 1.55. The only simple molecular ratios of the three oxides richer in lime in the possible area are $N_5C_4S_6$ and $N_2C_2S_3$. On preparation the former gave a mixture of NCS and the new compound. The latter, while containing a large proportion of the new compound, gave certain additional strong lines in a powder photograph, which did not appear in the pattern of

the 4:3:5 compound. The invariant point F is a little unusual in that the temperature falls along both boundaries FG and FI, and is therefore a "falling reaction point." No eutectic was found in the soda-rich part of the area investigated.

The presence of a C_3S_2 field is indicated by the appearance of a small liquidus for C_3S_2 in the lime-silica binary system. A series of quenches on charges of the composition 5% Na_2O, 50% CaO, 45% SiO_2, which lies near the edge of the C_2S field, shows that a long, narrow field extends to nearly 9% Na_2O. At the reaction point A C_3S_2 is in equilibrium with C_2S, NC_2S_2 and liquid, and at the eutectic B is in equilibrium with αCS, NC_2S_2 and liquid.

In table 3 are given the composition and temperatures of the nine invariant points.

TABLE 3

Ternary Invariant Points

Point	Phases	Type	Composition		Temp. °C.
			Na_2O %	CaO %	
A	C_2S-C_3S_2-NC_2S_2	Reaction	8.8	44.5	1280
B	α CS-C_3S_2-NC_2S_2	Eutectic	8.2	44.5	1270
C	NC_2S_2-α CS-NC_2S_2	Eutectic	12.5	39.5	1255
D	NC_2S_2-NCS-NC_2S_2	Eutectic	24.0	32.0	1260
E	C_2S-NCS-NC_2S_2	Eutectic	23.0	37.6	1440
F	NC_2S_2-NCS-$N_4C_5S_3$	Reaction	39.8	16.7	1130
G	NC_2S_2-N_2CS_3-$N_4C_5S_3$	Reaction	40.0	16.0	1120
H	NS-N_2CS_3-$N_4C_5S_3$	Reaction	45.0	10.5	990
I	NS-NCS-$N_4C_5S_3$	Reaction	48.0	10.5	950

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APPENDIX 1

Results of Quenching Experiments

	Composition Wt. %			Temperature	Phases
	Na_2O	CaO	SiO_2	°C.	Observed
The NC_2S_3 field	13.0	39.0	48.0	1286	Glass
				1281	Glass, rare NC_2S_3
				1275	Glass, NC_2S_3
	13.1	37.6	49.3	1287	Glass
				1280	Glass, NC_2S_3
				1255	NC_2S_3, NC_3S_5, tr. glass.
	14.0	38.0	48.0	1270	Glass, NC_2S_3
				1260	NC_2S_3, NC_3S_5, glass
	14.8	37.1	48.1	1280	Glass
				1266	Glass, NC_2S_3
	17.5	31.6	50.9	1285	Glass
				1283	NC_2S_3, rare glass
	17.6	34.1	48.3	1274	Glass
				1269	Glass, NC_2S_3
	20.0	30.0	50.0	1288	Glass, NC_2S_3
	22.1	31.4	46.5	1240	Glass
				1236	Glass, NC_2S_3

APPENDIX 1 (Cont.)

Results of Quenching Experiments

	Composition Wt. %			Temperature	Phases
	Na_2O	CaO	SiO_2	°C.	Observed
The NC_2S_3 field	23.0	30.4	46.6	1255	Glass
				1251	Glass, NC_2S_3
				1236	NC_2S_3, glass
				1228	NC_2S_3, NCS, tr. glass
	23.0	32.0	45.0	1266	Glass
				1258	Glass, NC_2S_3
	24.4	30.2	45.4	1269	Glass, rare NC_2S_3
				1258	Glass, NC_2S_3, NCS.
	29.6	25.1	45.3	1209	Glass
				1199	Glass, NC_2S_3
	31.0	26.7	42.3	1234	Glass
				1229	Glass, tr. NC_2S_3
	34.4	20.2	45.4	1149	Glass
				1141	Glass, NC_2S_3
	38.3	18.2	43.5	1115	Glass
				1101	Glass, NC_2S_3
The N_2CS_3 field	38.2	15.4	45.4	1128	Glass
				1117	Glass, N_2CS_3
	44.5	10.0	45.5	1010	Glass, rare N_2CS_3
				988	Glass, N_2CS_3, NS
				961	NS, $N_4C_2S_5$, N_2CS_3
The NCS field	24.1	35.4	40.5	1450	Glass
				1442	Glass, NCS
	28.0	29.8	42.2	1270	Glass
				1267	Glass, NCS
	29.5	31.2	39.3	1348	Glass
				1340	Glass, NCS
	29.7	30.1	40.2	1379	Glass
				1370	Glass, NCS
	31.0	31.3	37.7	1452	Glass
				1429	Glass, rare NCS

APPENDIX 1 (Cont.)

Results of Quenching Experiments

	Composition Wt. % Na_2O	CaO	SiO_2	Temperature °C.	Phases Observed
The NCS field	32.8	27.0	40.2	1321	Glass
				1315	Glass, NCS
	33.7	25.5	40.8	1312	Glass
				1290	Glass, rare NCS
	34.6	23.5	41.9	1208	Glass, rare NCS
	36.8	20.4	42.8	1153	Glass, rare NCS & NC_3S_2
				1143	NCS, NC_3S_2
	39.1	18.3	42.6	1167	Glass
				1157	Glass, rare NCS
	39.8	20.1	40.1	1317	Glass, rare NCS
				1290	Glass, NCS
	40.0	17.5	42.5	1145	Glass, NCS
				1124	Glass, NCS, $N_4C_3S_3$
	41.3	18.7	40.0	1159	Glass
				1139	Glass, NCS
				1096	Glass, NCS, $N_4C_3S_3$
	42.2	17.0	40.8	1120	Glass, rare NCS
				1091	Glass, NCS, $N_4C_3S_3$
	42.9	15.5	41.6	1065	Glass, NCS
				1005	Glass, NCS, $N_4C_3S_3$
	44.5	15.1	40.4	1136	Glass
				1125	Glass, NCS
				1003	Glass, NCS
				949	NCS, NS, $N_4C_3S_3$, glass
	47.0	13.0	40.0	1100	Glass, rare NCS
				1073	Glass, NCS
	49.0	10.0	41.0	942	Glass, rare NCS
				929	Glass, NCS, rare NS
				921	Glass, NCS, NS
The NC_3S_2 field	11.0	43.0	46.0	1289	Glass, NC_3S_2
				1268	Glass, NC_3S_2, rare CS

APPENDIX 1 (Cont.)

Results of Quenching Experiments

	Composition Wt. % Na_2O	CaO	SiO_2	Temperature °C.	Phases Observed
The NC_2S_3 field	11.0	44.0	45.0	1407	Glass
				1367	Glass, NC_2S_3,C_2S
	14.0	39.0	47.0	1272	Glass
				1267	Glass, NC_2S_3, C_2S_3
	15.0	38.0	47.0	1268	Glass
				1261	NC_2S_2, NC_2S_3, glass
	15.0	40.0	45.0	1347	Glass
				1332	Glass, NC_2S_2
	17.0	37.0	46.0	1310	Glass
				1301	Glass, NC_2S_2
	17.8	36.6	45.6	1303	Glass
				1300	Glass, rare NC_2S_2
	18.0	38.0	44.0	1384	Glass, rare NC_2S_2
	20.0	37.0	43.0	1389	Glass, rare NC_2S_2
				1366	Glass, NC_2S_2
	20.1	34.8	45.1	1315	Glass
				1307	Glass, NC_2S_2
	21.07	38.11	40.82	1446	Glass, rare NC_2S_2
				1442	Glass, NC_2S_2
				1435	NC_2S_2
	22.0	35.0	43.0	1370	Glass
				1353	Glass, NC_2S_2
	23.0	36.0	41.0	1417	Glass
				1402	Glass, NC_2S_2
				1386	Glass, NC_2S_2, NCS
	24.0	33.0	43.0	1327	Glass, rare NC_2S_2
The $N_4C_3S_2$ field	40.0	16.5	43.5	1124	Glass, rare $N_4C_3S_2$ & NC_2S_3
	41.0	16.0	43.0	1092	Glass, rare $N_4C_3S_2$
	41.5	15.0	43.5	1092	Glass, rare $N_4C_3S_2$

APPENDIX 1 (Cont.)

Results of Quenching Experiments

	Composition Wt.% Na_2O	CaO	SiO_2	Temperature °C.	Phases Observed
The $N_4C_2S_5$ field	43.2	15.0	41.8	1046	Glass
				1038	Glass, $N_4C_2S_5$
	43.3	14.7	42.0	1050	Glass, rare $N_4C_2S_5$
				1045	Glass, $N_4C_2S_5$
				945	Glass, $N_4C_2S_5$, NS
	44.0	13.3	42.7	1024	Glass, rare $N_4C_2S_5$
				1014	Glass, $N_4C_2S_5$
				974	Glass, $N_4C_2S_5$, NS
	45.6	10.3	44.1	989	Glass
				981	Glass, rare $N_4C_2S_5$ & NS
				968	Glass, $N_4C_2S_5$, NS
	47.0	12.0	41.0	1035	Glass
				1005	Glass, $N_4C_2S_5$
				950	Glass, $N_4C_2S_5$, NS, NCS
The NS field	46.0	10.0	44.0	983	Glass
				979	Glass, NS
	46.5	8.4	45.1	1012	Glass
				1006	Glass, NS
	47.0	10.0	43.0	976	Glass
				971	Glass, rare NS
				959	Glass, NS, $N_4C_2S_5$
	47.2	7.1	45.7	1031	Glass
				1019	Glass, NS
				984	Glass, NS, $N_4C_2S_5$
	48.0	10.0	42.0	967	Glass
				963	Glass, NS, NCS
	48.5	4.1	47.4	1065	Glass
				1058	Glass, NS, rare NCS
	51.0	2.0	47.0	1069	Glass
				1061	Glass, NS
The C_2S field	4.0	51.0	45.0	1438	Glass
				1426	Glass, C_2S
				1400	Glass, C_2S_2, α CS

APPENDIX 1 (Cont.)

Results of Quenching Experiments

	Composition Wt. %			Temperature °C.	Phases Observed
	Na_2O	CaO	SiO_2		
The C_2S field	4.9	55.1	40.0	1534	Glass, C_2S
	5.0	50.0	45.0	1392	Glass, rare C_2S
				1383	Glass, C_2S
				1364	Glass, C_2S, C_3S_2
				1334	Glass, C_2S_2, CS
				1278	C_3S_2, α CS, glass
	7.9	51.2	40.9	1509	Glass, C_2S
	9.0	45.0	46.0	1324	Glass, rare C_2S
				1307	Glass, C_2S, NC_3S_2
	10.0	45.0	45.0	1341	Glass, rare C_2S
				1338	Glass, C_2S
	17.5	42.3	40.2	1453	Glass
				1449	Glass, C_2S
	20.0	40.0	40.0	1460	Glass, C_2S
				1442	C_2S, glass
	23.2	46.1	30.7	1580	C_2S, NCS, glass
	27.1	42.8	30.1	1585	C_2S, rare NCS, glass
The α CS field	9.0	44.0	47.0	1340	Glass
				1339	Glass, α CS
	12.0	37.0	51.0	1326	Glass, α CS
	12.1	38.7	49.2	1296	Glass, α CS

APPENDIX 2

X-ray Diffraction Lines of NC_2S_2 and $N_4C_3S_5$

d in Å

NC_2S_2				$N_4C_3S_5$			
d	I	d	I	d	I	d	I
4.04	W	1.836	S	6.71	VW	2.171	VW
3.86	W	1.814	W	5.20	VW	2.030	S
3.57	M	1.717	W	4.76	VW	1.972	W
3.29	W	1.619	W	4.41	VW	1.928	VW
3.15	W	1.599	M	4.07	M	1.889	W
2.90	W-M	1.586	W	3.98	S	1.831	S
2.76	VS	1.554	W	3.65	VW	1.803	VW
2.71	VS	1.536	W	3.52	W	1.774	VW
2.61	S	1.514	M-S	3.36	W	1.738	VW
2.59	S	1.371	W-M	3.29	W	1.714	W
2.50	VW	1.350	W-M	3.21	W	1.616	W
2.41	W	1.327	VW	2.98	VW	1.594	M
2.31	W	1.300	W	2.93	W	1.548	W
2.27	W	1.289	W	2.88	S	1.536	W
2.22	W	1.276	W	2.78	W	1.515	M
2.18	W	1.263	W	2.73	S	1.368	W
2.11	VW	1.216	W	2.70	S	1.351	M
2.04	S	1.116	W	2.60	M	1.330	VW
1.966	VW	1.085	W	2.56	M	1.303	VW
1.922	VW	1.018	W	2.46	W	1.278	VW
1.866	W			2.41	S	1.266	W
				2.251	VW	1.165	W
				2.219	VW	1.089	W

[American Journal of Science, Vol. 251, August 1953, Pp. 602-605]

RUIN-MARBLE: A STYLOLITIC FORMATION?

B. M. SHAUB

ABSTRACT. Ruin-marble, a banded calcareous material, is described as a structure related to stylolites in which the many column-like projections are cross sections of more or less small tabular fault blocks which have originated through contraction and pressure phenomena accompanying the dewatering of the calcareous sediments. This structure indicates the possibility that stylolites can possibly develop under certain conditions without the presence of a clay seam. There is no evidence that solution has occurred at any time during the development of the ruin-marble. The more highly ferruginous layers remained more plastic than the calcareous layers and were occasionally pressed into the fractures and other places where there was a sufficient release of pressure.

RUIN-MARBLE is known chiefly from the environs of Florence, Italy, and also from Klosterneuburg, Austria. It is usually displayed in mineral collections along with such materials as landscape agate and other chalcedonic structures which contain included dendritic configurations of various kinds. In their manner of origin and their appearances there is, however, a very great difference. Little information appears to be available in the literature regarding ruin-marble. Dana (1889, p. 679) states that "ruin-marble is a kind of compact calcareous marl, showing when polished, pictures of fortifications, temples, etc. in ruins due to infiltration of oxide of iron; . . . " Merrill (1910, p. 340) describes ruin-marble as a "very compact yellowish or drab limestone, the beds of which appear to have been fractured in every conceivable direction by geological agencies, after which the resultant fragments have become cemented by a calcareous or ferruginous cement. The rock is therefore really a breccia, although the proportional amount of cement is very small, and the actual displacement of the various particles but slight. When cut and polished the slabs have somewhat the appearance of mosaics representing the ruins of ancient castles or other structures. Hence the name of 'ruin-marble'."

Because of the lack of suitable material to conduct a more comprehensive study of the structure, a tentative description will be given in order to bring again this unusual structure to the attention of those interested in sedimentary processes and

the structures developed therein during the period of dewatering and consolidation.

Examination of the photographs (plate 1) shows the presence of certain prominent dark layers, rich in iron oxide, which carry across a number of the "columns" in the same stratigraphic sequence, while in others the same prominent dark bands appear to be missing and a new sequence of banding appears. One of the prominent features is the pronounced wedging of the columns, showing not only that the relative movements have been up and down with lateral displacements to the right or left but also there have been similar displacements to the front and rear of the sections shown in the photographs. A small piece of material from Klosterneuburg, Austria, was polished on three planes at approximately right angles, and in each plane components of the displacements were clearly shown to have occurred in two directions in each plane. The boundaries between the various parts are in reality minute fault planes, the continuation of which can often be traced into the "overlying" unbanded material, according to the orientation of the illustrations. In the few pieces examined there were no open cavities along these fault planes. This is probably due to the retention by the material of a sufficiently high degree of plasticity to prohibit the formation of cavities at this stage; or any cavities encountered during the preparation of the material for market would probably have been discarded.

It is quite evident from the illustrations that the iron oxide bands retained a higher degree of plasticity during the period of the pronounced adjustment, for they were frequently squeezed into bulges and along the many small fault planes as shown at a, b, c, d, and e in figure 1 and at m, n, o, and p in figure 2. There are also instances of slight bending of the bands along the contacts, but in no more than a few instances is the bending prominent. This shows that there was no heavy lateral pressure along the small fault planes at the time of displacement and that the development progressed slowly under very low differential pressures continually regenerated here and there through the structure during the dewatering process.

It is, indeed, certain that there has not been any solution of the calcareous material before, during, or after the formation of the structure, for there is not the slightest indication that there has been any removal of material by solution.

The origin of the structure appears to be a simple straightforward process which consisted of shifting small masses of the banded calcareous material from place to place through the combined action of the contraction and slight differential pressures during the period of contraction during dewatering. The pressures operating were a combination of the negative contraction forces and the static pressure due to the overlying material. The contractions produced a lateral relief of pressure which permitted the interfingering of the various columns as they now appear. They are in reality quite irregular tabular fault blocks of minute proportions. While the process is a simple one, the relative movements of the pieces are very complex and intricately interrelated to one another. Sections may appear to be missing but they have been displaced to the front or rear of the section in view.

This structure appears to be closely related to stylolitic development. Its unique characteristics have been inherited through its peculiar sedimentary environment. There are, however, some striking differences between ruin-marble and the various stylolitic seams and columns. One feature of the ruin-marble which first attracts attention is the clearcut sharp angular outlines of the blocks or sections which are in marked contrast to the more or less continuous nature of the material on the two sides of a normal stylolite seam. It is the sharp and pronounced banding which shows the relative movements of the blocks in the lower part of the illustration. However, the non-banded part above has probably undergone a similar adjustment as the fracture planes continue into this material. A few waterlines appear to cross all fault blocks at the same level. A similar adjustment, but probably of a more uniform, continuous nature occurs on the two sides of most of the normal stylolite seams, for the Tennessee marble *carries a very great number of indications of plastic adjustment in the mass of the marble on the two sides of the seams.*

The striking difference between the ruin-marble and the normal stylolite seam is the absence of a clay seam which usually accompanies and is an essential to the normal stylolite seam, although the clay partings may be almost *infinitesimally thin* at times. The absence of clay seams here shows that under certain conditions they are probably non-essential in the development of stylolites. The movement of the residual water into

PLATE I

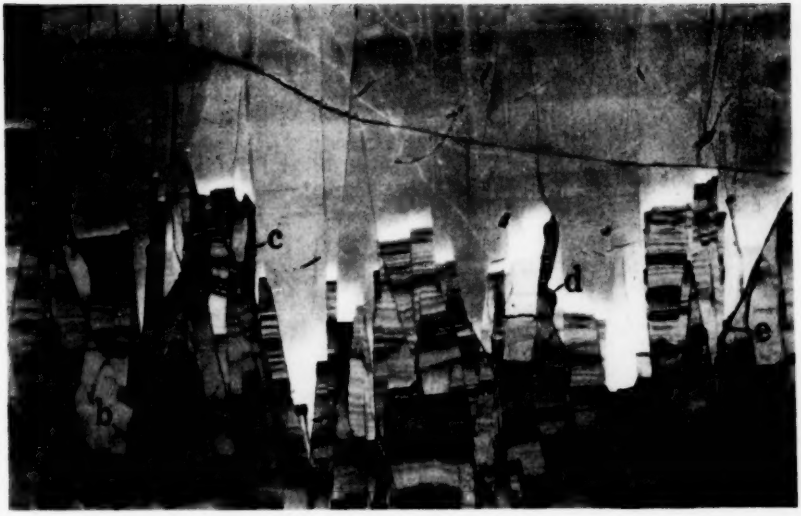

Fig. 1. Ruin-marble from Florence, Italy.

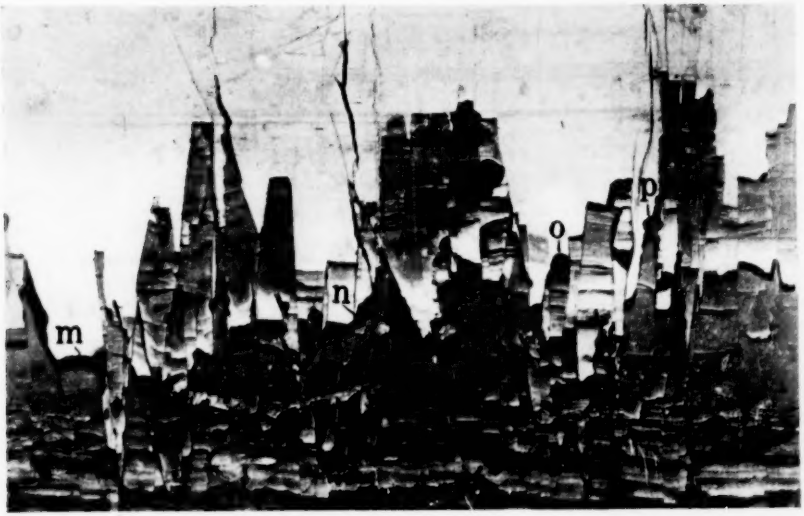

Fig. 2. Ruin-marble showing a very intricate rearrangement of the banded layers of calcareous material. Locality unknown.

and along a stylolite seam after its development by plastic flow can deposit the minute amount of colloidal clay-like material often found along such seams.

Acknowledgment.—The writer wishes to express his appreciation to Mr. Howard E. Norris, curator of the Henry Miller mineral collection on exhibit at the Loomis School at East Windsor, Connecticut, for the privilege of examining and photographing the ruin-marble specimen from Florence, Italy.

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DOWNSTREAM CHANGE OF VELOCITY IN RIVERS

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ABSTRACT. Because river slope generally decreases in a downstream direction, it is generally supposed that velocity of flow also decreases downstream. Analysis of some of the large number of velocity measurements made at stream-gaging stations demonstrates that mean velocity generally tends to increase downstream. Although there are many reaches in nearly all rivers where mean velocity decreases downstream, the general tendency for conservation or for downstream increase was found in all data studied.

Computations of bed velocity indicate that this parameter also tends to increase downstream.

Near the streambed, shear in the vertical profile of velocity (rate of decrease of velocity with depth) tends to decrease downstream. This down-valley decrease of shear implies decreasing competence downstream.

General Statement

BECAUSE rivers tend to be steep in their headwaters and flow at flat gradients near their mouths, it appears logical to suppose that the stream velocities are great in the headwaters and decrease downstream.

Yet, the engineer who actually measures stream velocities for purposes of computing the water discharge uses the same current meter for a measurement in a small brook as in the Mississippi River. The velocities that he measures are not sufficiently different in small headwaters and downstream reaches to furnish any obvious indication of a definite pattern of organization in the data.

The purpose of the present paper is to organize some of the large amount of available data on river velocities so that the downstream change may be evaluated.

DATA ON RIVER VELOCITY

There are more than 6000 gaging stations in the United States. Records on many streams are more than 50 years in length. At each station periodic measurements of discharge are made by use of a current meter. The discharge on other days is determined indirectly from the recording chart or daily readings of stage (water surface elevation).

When a river measurement is made by current meter, the water speed is measured at about 20 to 30 verticals across the section and usually at two depths in each vertical. From these direct measurements the form of the river cross section may be plotted, as well as an approximate pattern of the vertical and horizontal distribution of velocity.

In stream-gaging practice, the mean velocity is not a simple average of the velocity measurements made at various positions across the stream. The current meter placed in a given position in the cross section measures the downstream movement of the water in a given interval of time. The metering thereby defines a prism of water whose width and depth are known and whose downstream dimension is represented by the measured movement. The volume of the prism that passed the meter during the measured time interval represents the discharge in units of volume per unit time as denoted by the identity,

$$Q \equiv aV$$

where Q is discharge in cubic feet per second, a is cross-sectional area in square feet, and V is velocity in feet per second. The total discharge past the cross section is the sum of the discharges represented by the individual prisms.

The total discharge having been obtained in this way, the mean velocity used in hydraulic practice is an expression of the velocity of a single rectangular prism having the same cross-sectional area as the stream. This mean velocity is defined as the quotient of the discharge in the cross section divided by the cross-sectional area of flowing water. Arithmetically the mean velocity so defined is equivalent to a weighted mean in which the velocities of the individual prisms are weighted by the cross-sectional area of the respective prisms.

Owing to the distribution of the velocity with depth, the mean velocity is closely equal to that existing at a position 0.6 of the distance from the water surface to the streambed.

It is recognized (Rubey, 1938, p. 129) that the mean velocity in the cross section is not the most meaningful velocity with respect to sediment transport. Direct measurements of the bed velocity, however it may be defined, are not possible at present. On the other hand, a large fund of measurements of mean velocity exists. It appears instructive to examine the downstream change of that velocity parameter for which meas-

urements exist, recognizing that the parameter, mean velocity, is not ideal for discussing geomorphic progresses. In a later section, downstream changes of a computed bed velocity will be presented.

As indicated previously, velocity in a stream is a conservative quantity. Even the maximum point velocity (maximum at any point in the cross section) does not ordinarily greatly exceed the mean velocity for the cross section. An unpublished study by the Geological Survey of 2950 measurements of maximum point velocity values from a variety of rivers showed a median value of 4.11 ft per sec., mean of 4.84 ft per sec. and less than one per cent of the total exceeded 13 ft per sec.

The largest value of maximum point velocity in a natural river channel ever measured by stream-gaging personnel of the U. S. Geological Survey was about 22 ft per sec. Approximately this value was measured on several occasions: Brazos River at Waco, Texas, Sept. 27, 1936; Canadian River near Newcastle, Okla., May 4, 1941; New River at Caperton, W. Va., Aug. 1940; Potomac River at Washington, D. C., May 14, 1932.

FLOW DURATION AND FLOOD FREQUENCY

In order to compare the mean velocity in a small headwater tributary with that in a large trunk stream it is necessary to establish a basis of comparability. It would be meaningless to compare the velocity during a flood flow at one point on a river with that of a low flow at some other point. The two points to be compared must both be experiencing flood flow or must both be experiencing low flow. The relative magnitude of flow is discussed in hydrologic practice by the use of the flood-frequency concept. If the mean flow for each day of record at a station is tallied in categories of size, the number of days on which each category of size is equaled or exceeded may be counted. The number of days in each category may be accumulated from the largest category to the smallest, and divided by the total number of days. The resulting quotients represent the percentage of the total days each given discharge is equaled or exceeded. When the discharge equaled or exceeded is plotted against per cent of time the resulting curve, called the "flow-duration curve," is directly analogous to the plot used in sedimentation work to describe the size-frequency distribution of a sediment sample.

The velocity at two stations along a river system may be logically compared when the discharge at each represents the same percentage of time equaled or exceeded.

The flow-duration curve adequately describes the per cent of time a given mean daily flow is exceeded. A flood peak, however, may last only a few hours and therefore would not be represented by the mean flow for the day on which it occurs. The frequency of flood peaks, therefore, is usually determined by the use of a series of data representing the momentary peak-discharge values for each year of record. This series is arranged in order of magnitude, and plotted in terms of the recurrence interval in years versus the peak discharge.

There are minor variations in methods of preparing flow-duration and flood-frequency curves, and procedures for analysis are adequately described in the literature (Jarvis, 1936, pp. 68-85; Linsley, Kohler, and Paulhus, 1949, pp. 182-198), but the general principles underlying all methods are essentially similar. In the analysis contained herein, the procedures for obtaining flow-duration and flood-frequency curves were those in current use in the U. S. Geological Survey.

MEAN VELOCITY AT A GIVEN RIVER CROSS SECTION

The mean velocity in a river cross section tends to increase with discharge. This becomes evident from a consideration of a relation widely used in hydraulic practice, the Manning equation, expressing the balance of frictional losses (which are dependent on velocity, hydraulic radius, and roughness) and the slope of the energy grade line.

$$V = 1.5\, R^{2/3}\, \frac{s^{1/2}}{n} \qquad (1)$$

where V = mean velocity

$R = \dfrac{\text{area of cross section}}{\text{wetted perimeter}}$ = hydraulic radius

s = slope of the energy grade line, equal to slope of water surface for steady, uniform flow

n = roughness parameter

For most natural river channels the width is great compared with depth and the hydraulic radius, R, is approximately equal

to the mean depth, d. The discharge, Q, is expressed as the product of mean velocity and cross-sectional area,

$$Q \equiv aV$$

With increasing discharge in most natural river cross sections, the water-surface slope remains nearly constant and roughness tends to decrease. Thus velocity tends to increase with increasing depth and therefore with increasing discharge.

Figure 1 presents a typical relation of mean velocity to discharge in a river cross section. The data are representative of the large volume of observational material collected at stream-gaging stations by the use of current meter. Such data are available in the files of the Water Resources Division, U. S. Geological Survey. For the station pictured, the mean annual discharge during a 9-year period of record was 881 cfs, at which the corresponding velocity is 3.2 ft per sec. At any given discharge a corresponding mean velocity may be read off the graph.

CHANGE OF MEAN VELOCITY DOWNSTREAM

It has been demonstrated (Leopold and Maddock, 1953) that the velocity corresponding to mean discharge tends to increase with increasing discharge downstream.

The reason for the downstream increase of velocity with increasing discharge has been previously discussed in detail (Leo-

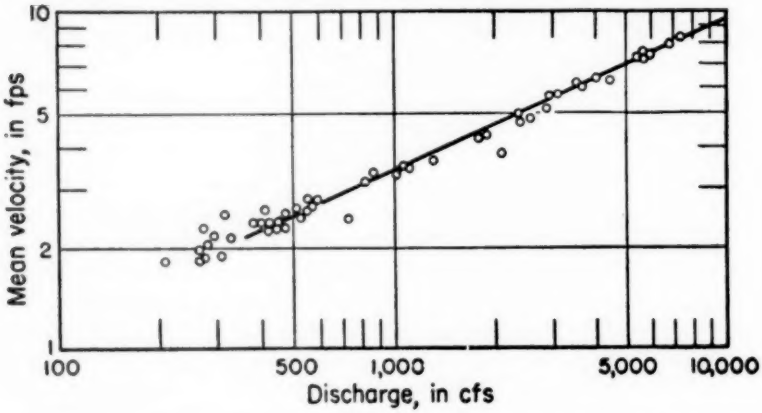


Fig. 1. Relation of mean velocity to discharge in a given river cross section, Stillwater River near Absarokee, Montana.

pold and Maddock, 1953, pp. 15, 42, and 43). The explanation can be summarized by saying that the downstream increase of depth overcompensates for the decrease in slope.

In that study, the river data considered in analyzing the downstream change of velocity did not include enough high mountain streams of very steep gradient and in which caliber of load rapidly decreased downstream. Moreover, downstream changes in bed velocity were not discussed. The present paper reviews the evidence previously discussed, presents data for streams representing a more stringent test of the generalization, and discusses similar relations involving bed velocity.

The mean annual discharge at a given station is the arithmetic mean of all the daily discharge values during the period of record. In most river systems, the mean discharge increases downstream owing to the increase in drainage area in the downstream direction. As a general rule, mean discharge increases directly with some power (near unity) of drainage area in a given basin. Though there are many basins, particularly in the western United States, in which mean discharge decreases owing to irrigation diversion and to channel losses, in most basins there is an increase downstream.

The mean discharge at most points along a river system occurs somewhat less often than half the time. The per cent of time the mean discharge is equaled or exceeded is not constant but is sufficiently conservative to allow a comparison of upstream and downstream points. Because it is a parameter readily available for a large number of gaging stations, its use allows the comparison of a considerable number of stations. The computation of complete flow-duration and flood-frequency curves has been completed for a smaller number of selected stations chosen for a more critical evaluation.

The mean velocity at a number of stations along a river length will be compared at discharges corresponding to their respective mean annual values. Figure 2a presents a sample of this relation for a small portion of the upper Missouri River basin and a few of stations on the Missouri and Mississippi rivers.

Each point in figure 2a represents a particular gaging station and its abscissa value corresponds to its mean annual discharge. Within each tributary basin, the points are numbered in sequence from downstream toward the headwaters. In nearly all

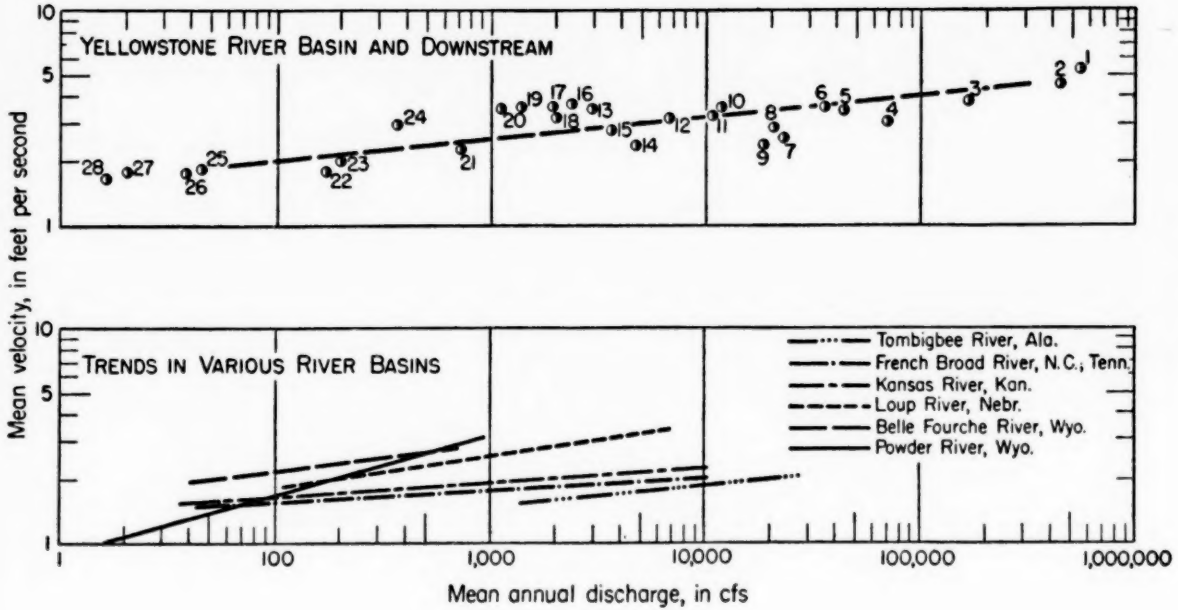
basins the mean annual discharge increases downstream but there are examples, such as the Greybull River, in which the mean discharge at a downstream station, Basin, Wyoming (point 23), is less than at Meeteetsee farther upstream (point 24), owing to irrigation diversion and channel losses.

Stations in the Yellowstone and Bighorn River basins chosen to represent a headwater area have velocities at mean annual discharge of 2 to 4 ft per sec., whereas the lower Missouri and Mississippi rivers have corresponding velocities between 4 and 6 ft per sec. The mean line in the graph of figure 2a has been fitted by inspection and is intended merely to call attention to the definite tendency of the points to progress toward higher velocity values downstream.

Fig. 2. Change of mean velocity as mean annual discharge increases downstream. In 2a above, each point represents a gaging station on Yellowstone River system and downstream. In 2b, mean trend lines for various river basins are drawn.

Identification of Stream-gaging Stations in Figure 2A

1. Mississippi R. at Vicksburg, Miss.
2. " " Memphis, Tenn.
3. " " St. Louis, Mo.
4. Missouri R. at Hermann, Mo.
5. " " Kansas City, Mo.
6. " " St. Joseph, Mo.
7. " " Pierre, S. D.
8. " " Bismark, N. D.
9. " " Williston, N. D.
10. Yellowstone R. nr. Sidney, Mont.
11. " " Miles City, Mont.
12. " " Billings, Mont.
13. " " Corwin Springs, Mont.
14. Bighorn R. at Custer, Mont.
15. " " St. Xavier, Mont.
16. " " Kane, Wyo.
17. " " Thermopolis, Wyo.
18. " " Manderson, Wyo.
19. Wind R. nr. Crowheart, Wyo.
20. " " Riverton, Wyo.
21. " " Burris, Wyo.
22. " " Dubois, Wyo.
23. Greybull R. at Basin, Wyo.
24. " " Meeteetsee, Wyo.
25. Owl Creek nr. Thermopolis, Wyo.
26. Medicine Lodge Cr. nr. Hyattville, Wyo.
27. Gooseberry Cr. nr. Grass Creek, Wyo.
28. N. Fk. Owl Cr. nr. Anchor, Wyo.


YELLOWSTONE RIVER BASIN AND DOWNSTREAM
TRENDS IN VARIOUS RIVER BASINS
Tombigbee River, Ala.
French Brood River, N.C., Tenn.
Kansas River, Kan.
Loup River, Nebr.
Belle Fourche River, Wyo.
Powder River, Wyo.
Mean velocity, in feet per second
Mean annual discharge, in cfs
10
5
1
100
1,000
10,000
100,000
1,000,000

In any given reach of river the velocity may decrease downstream, as can be seen by a study of figure 2a. Also there is a considerable scatter of points in the graph. But for present purposes attention is directed to the general trend which is toward increasing velocity downstream.

In figure 2b the trend lines of velocity versus mean annual discharge are plotted for a number of rivers chosen to represent a variety of physiographic provinces. The general tendency for increasing velocity as mean annual discharge increases downstream is well exemplified.

The mean annual discharge in many streams is equaled or exceeded about one out of every 4 days in a long period. Inasmuch as the greatest total sediment load is carried by a stream during high or flood flows, it is of interest to examine the trend of velocity at discharge rates less frequent than the mean annual rate. For this purpose figure 3 presents the trend lines for discharges equaled or exceeded various percentages of time from 50 per cent to 1 per cent, in the Maumee and Scioto River basins in Ohio. The two basins were considered together in order to be able to utilize a sufficient number of points to define the downstream trends. The Maumee and Scioto basins were chosen because complete flow-duration curves for stations in Ohio are available in the study by Cross and Bernhagen (1949). Figure 3 is taken from Leopold and Maddock (1953).

The scatter of points about the lines represented in figure 3 was again great, but the general trend of increasing velocity

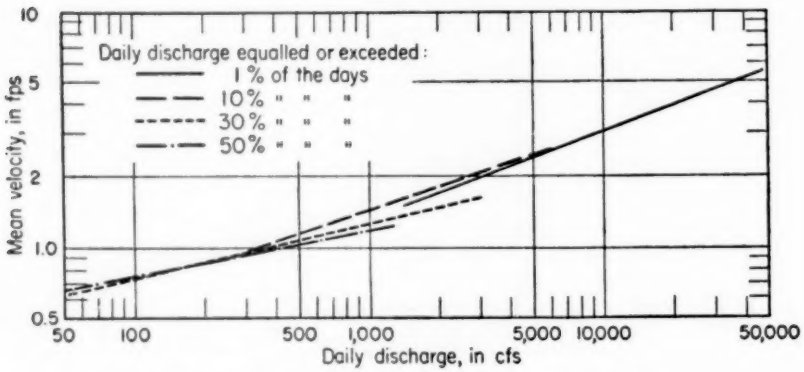


Fig. 3. Relation of mean velocity to daily discharge equaled or exceeded various percentages of time, Maumee and Scioto river basins, Ohio.

downstream in these basins appears to characterize each of the duration percentages analyzed.

But the Maumee and Scioto basins still do not furnish a critical test of how mean velocities vary downstream because the topographic relief in those drainage basins is not great. Moreover, the materials composing the beds of those rivers are relatively fine grained, consisting for the most part of clay, silt, and sand. It is logical to suppose that if in any stream the velocity tends to decrease downstream, such a stream would be one that is very steep in the headwaters and flat downstream. Moreover, it would be a stream the bed of which consists of coarse gravel in the headwaters, decreasing in grain size to sand downstream. Furthermore, the graphs presented earlier were for discharges exceeded no less often than 1 per cent of the days, or one day in 100. It might be supposed that the more extreme floods, which are the most effective in moving coarse gravel in the headwaters, would be the flow condition most likely to show a tendency for decreasing velocity downstream.

As an example of these stringent conditions a series of tributaries of the upper Yellowstone River in Wyoming and Montana were selected. The streams chosen include the Greybull River, Rock Creek, the Clarks Fork, and some of their headwater tributaries. These streams drain into the Bighorn River, a tributary to the Yellowstone River, or directly into the Yellowstone. These headwater tributaries carry coarse gravel which decreases in caliber rapidly downstream. The slopes of the uppermost tributaries are up to 100 feet per mile. More than one tributary to the Yellowstone was selected in order that a sufficient number of gaging stations would be included to provide an adequate test.

In accordance with good hydrologic practice, the flood-frequency curves for all gaging stations were developed for an identical period of record, 1923-1950, inclusive, and the records shorter than this were adjusted to the long record.

The flood data utilized at each station consisted of the peak or momentary discharge for each year of available record. Flood-frequency curves were developed for recurrence intervals of 5, 10, 25, and 50 years. By definition, a flood of 50-year recurrence interval means the momentary discharge equaled or exceeded on the average once in 50 years over a long period.

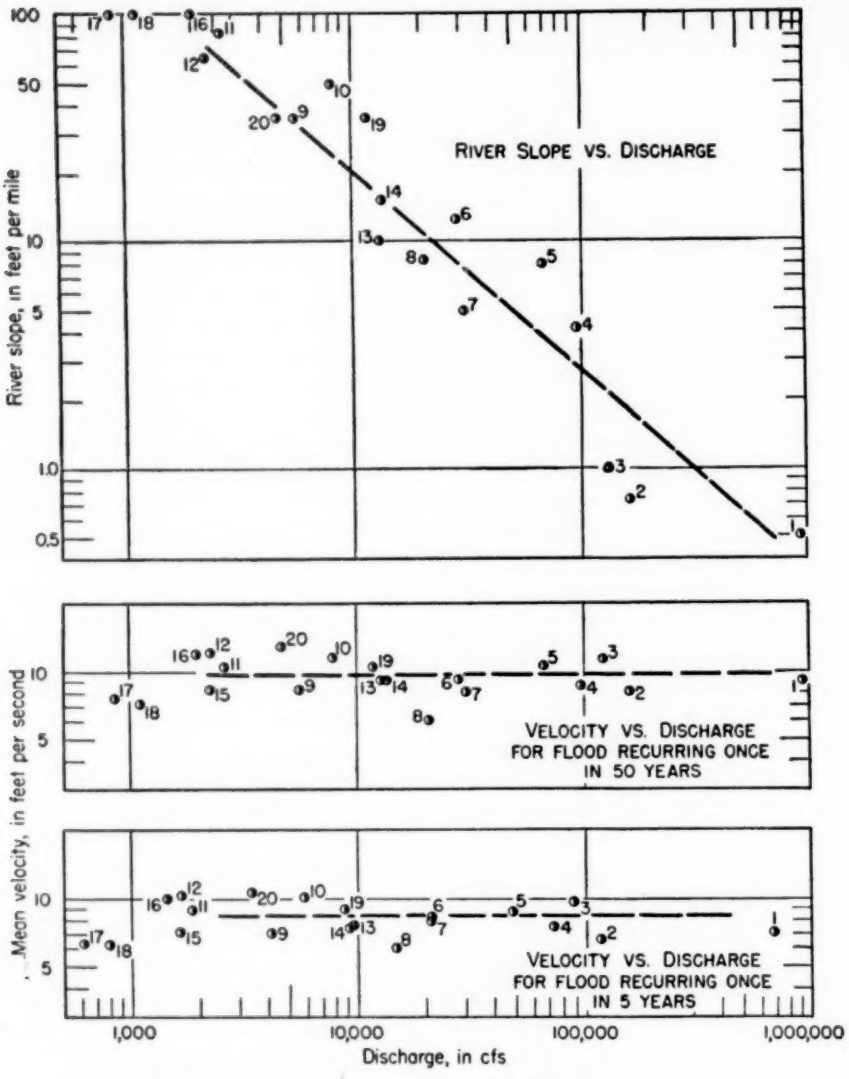


Fig. 4. Change of mean velocity with increase of discharge downstream for flood flows, Yellowstone River Basin and downstream. Upper figure shows how river slope decreases downstream. Lower figures show downstream change of velocity for 50 year and 5 year floods.

The relation of mean velocity to discharge was plotted for each station as in the example of figure 1. After the discharge for each recurrence interval chosen had been determined from the flood-frequency curves, the velocity corresponding to that discharge was read from the velocity-discharge curves.

The curves for recurrence intervals of 5 years and 50 years are reproduced in figure 4 for the stations in the upper Yellowstone basin. For comparison, a few stations on the upper Missouri River were included in the study, and one station on the main stem of the Mississippi River.

Dashed lines have been fitted by eye through the points in the velocity-discharge graphs of figure 4 merely to indicate the trend. Though there is scatter to the points, this indicates a remarkable conservation of the mean velocity downstream, and there is no definite tendency to decrease downstream in these rivers. A valid interpretation of these data is that in these streams mean velocity is essentially constant downstream.

The graph of river slope in feet per mile plotted against discharge (50-year recurrence interval), which is presented for comparison with the velocity-discharge graphs, shows the marked decrease of gradient downstream from 100 feet per mile to about half a foot per mile at the stations studied. That the velocity should remain essentially constant despite

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Identification of Stream-gaging Stations Shown by Points in Figure 4

1. Mississippi R. at St. Louis, Mo.
2. Missouri R. at Williston, N. D.
3. Yellowstone R. nr. Sidney, Mont.
4. " at Miles City, Mont.
5. " at Billings, Mont.
6. " at Corwin Springs, Mont.
7. Bighorn R. at Kane, Wyo.
8. " at Thermopolis, Wyo.
9. Greybull R. nr. Basin, Wyo.
10. " at Meeteetsee, Wyo.
11. " nr. Pitchfork, Wyo.
12. Wood R. at Sunshine, Wyo.
13. Clarks Fork at Edgar, Mont.
14. " at Chance, Mont.
15. Rock Cr. at Joliet, Mont.
16. " nr. Red Lodge, Mont.
17. W. Fk. Rock Cr. below Basin nr. Red Lodge, Mont.
18. Red Lodge Cr. above Reservoir nr. Boyd, Mont.
19. Stillwater R. nr. Absarokee, Mont.
20. Rosebud Cr. nr. Absarokee, Mont.

such a decrease in slope, and under the conditions of extreme flood in streams of rapidly decreasing bed-material size, is of interest.

Whether further study of additional data will discover streams in which the mean velocity rapidly decreases downstream is unknown, but no such trend has been found in any of a large number of streams studied by the author.

CHANGES DOWNSTREAM IN THE VELOCITY NEAR THE STREAMBED

Though the velocity near the streambed cannot at present be measured directly, the theory of vertical velocity distribution in pipes and open channels is sufficiently well developed that computations of velocity near the bed are possible. Though these computed velocities are approximate, they allow, nevertheless, an indication of the downstream trends which are under discussion here.

The velocity distribution in the vertical tends to follow a logarithmic law down to a position very close to the bed itself where the flow is laminar, and within the laminar sublayer the vertical changes are linear.

Bed velocity has been defined in a number of ways. Rubey (1938) considered it to be the velocity at the laminar boundary. Einstein (1950) discussed the movement of bed load in terms of the velocity in the "bed layer" which he found to be about 2 grain diameters in thickness.

The theory of turbulent flow derived from the study of pipe flow was applied to wide rivers by O'Brien (1937). Langbein (1940) further developed this concept and modified the O'Brien equations to a form which relates the velocity at a given distance above the bed (but outside the laminar sublayer) to the mean velocity. As O'Brien had previously done, Langbein showed that actual measurements of velocity distribution gave results similar to those from the O'Brien equation. On the basis of measurement data, Langbein presents a dimensionless graph of the velocity distribution in the vertical which related $F(z/d)$ to (z/d) where

$$F(z/d) = \frac{V - V_m}{\sqrt{gds}} \qquad (2)$$

V is the velocity at distance z from the bed, d is the depth, s is energy slope, V_m is the mean velocity in the vertical, and

$F(z/d)$ signifies a function of (z/d). Values of the function $F(z/d)$ may be read off Langbein's graph for given values of z and d. For concurrent values of V_m, d, and s representing a given cross section, the vertical velocity distribution may be computed.

Leopold and Maddock (1953) have studied the changes in depth, mean velocity, and water-surface slope in the downstream direction for numerous rivers. The downstream changes have been expressed as functions of discharge of a given frequency. From their data, which were in a form similar to those presented in figure 2b of the present paper, representative curves of the downstream changes of V_m, d, and s as functions of mean annual discharge have been chosen and are defined by the mutually consistent equations below. These relations may be considered to describe the characteristics of a typical midwestern river, though no single river is exactly like the composite represented by the equations.

$$d = 0.16\,Q^{0.40} \tag{3}$$

$$V_m = 1.3\,Q^{0.10} \tag{4}$$

$$s = 0.021\,Q^{-0.49} \tag{5}$$

where d is depth in feet, V_m is mean velocity in feet per second, s is water-surface slope (approximately equal to energy slope), and Q is mean annual discharge in cfs.

For a given value of Q, values of V_m, d, and s provided by equations 3-5 were substituted in equation 2, and using Langbein's graph, the velocity V at different distances z above the bed was computed. The computed velocities at three distances above the bed are shown in table 1 for several discharge values representing progressive downstream changes. Complete vertical-velocity curves for four of these assumed discharges are plotted in figure 5.

The four diagrams in figure 5 represent plots of velocity against stream depth at four places along the length of the assumed composite river. The logarithmic decrease of velocity with depth in a given cross section is apparent from the shape of the curves. The Langbein empirical relation between $F(z/d)$ extended only to z/d value of 0.1 and had to be extrapolated somewhat to compute the values of velocity near the bed for the deeper sections, but because his relation was a straight line

TABLE 1

Computed Values of Velocity and Shear near Streambed
Representing Changes in Downstream Direction

Depth d (ft)	Discharge Q (cfs)	Slope s	Mean velocity V_m (ft/sec)	Velocity in ft/sec at various distances above streambed (0.02 ft)	(0.05 ft)	(0.1 ft)	Shear between 0.05 and 0.1 ft (ft/sec/ft)
0.4	10	0.0070	1.6	1.1	1.34	1.52	3.6
1.0	100	.0023	2.0	1.3	1.54	1.71	3.4
2.0	580	.00098	2.4	1.6	1.82	1.97	3.0
3.5	2400	.00048	2.7	1.9	2.10	2.24	2.8
5.0	5800	.00030	3.0	2.1	2.32	2.45	2.6

on logarithmic paper and parallel to the O'Brien theoretical relation, the extrapolation appeared to be justified.

The measured velocities in the canal data used by Langbein gave the average position of the mean velocity at 0.67 the distance from the water surface to the streambed, whereas in most rivers it usually occurs at about 0.60 the depth. This slight difference does not materially affect the shape of the vertical-velocity curves.

It can be seen in both figure 5 and table 1 that, progressing downstream, depth increases faster than does mean velocity. Moreover, the velocity at any specific distance above the bed also increases downstream, as can be seen in the table. In the case of the particular river characteristics used here as typical, the downstream rate of increase of velocity 0.02, 0.05 and 0.10 foot above the bed is about the same as the downstream rate of increase of mean velocity.

On figure 5 dashed lines have been drawn between the points representing the velocity at 0.05 feet above the bed and the mean velocity. The slope of the dashed line then represents the shear $\frac{\Delta V}{\Delta d}$ between these two depths. The changing slope

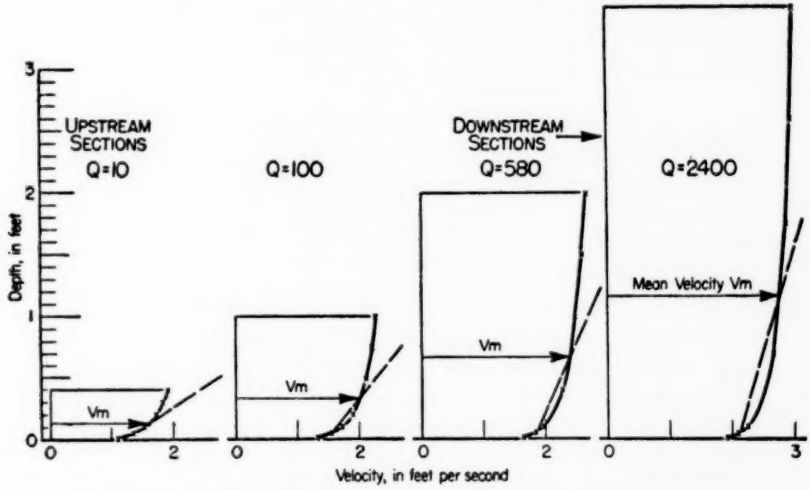

Fig. 5. Vertical velocity distribution computed for four sections representing progressive positions downstream in a river. Note that velocity near the bed as well as mean velocity increases downstream. Dashed lines connect velocity vectors at 0.05 feet above bed and the mean velocity.

of these dashed lines from the upstream section to the downstream section shows that this shear decreases downstream. In table 1, values of the shear between points 0.05 and 0.1 feet above the bed are listed and it can be seen that these shear values also decrease downstream.

The finite values of shear indicated by the righthand column in table 1 are presented only to provide a better visual picture of the physical nature of shear stress in the vertical profile of velocity. They represent merely examples of shear quantities and it is not implied that these are particularly important depths through which to measure shear.

The total shear on the bed, τ, is proportional to the product of slope and hydraulic radius; for wide channels where hydraulic radius is approximately equal to mean depth, $\tau \propto ds$. As indicated by the equations (3) and (5) for the composite river discussed earlier and for many of the individual rivers studied, slope decreases downstream slightly faster than depth increases. Thus the product depth x slope, to which total bed shear is proportional, decreases slightly downstream.

It appears, then, that in numerous rivers and in a synthetic composite river described here, total shear on the bed and shear between different levels near the bed decrease in the downstream direction. This decrease may obtain despite a downvalley increase of mean velocity and of velocity near the bed.

RELATION TO COMPETENCE AND LOAD

It is well known that the particle size of material composing the bed of a river decreases downstream owing to abrasion and sorting, and that there is a concomitant decrease in slope. It has long been supposed, therefore, that the velocity of a stream decreases downstream.

The reasoning on which this supposition is based may be summarized as follows: The downstream decrease in caliber of load caused by abrasion and sorting means that a smaller velocity is required to carry the small particles near the stream mouth than the large particles in headwater reaches. The smaller velocity required downstream is attained by progressive flattening of the slope downstream. If, in any reach, the contributed load is of such large caliber that the existing velocity is insufficient to move it, deposition occurs until the slope through the reach of deposition is increased sufficiently to produce a velocity great enough to move the particles.

The present analysis indicates that the velocity near the bed, as well as the mean velocity, tends to increase downstream. It has been shown, furthermore, that even under conditions of extreme flood in steep mountain streams the mean velocity does not decrease downstream. It is necessary, then, to revise our concept of how the mutual adjustment of slope, roughness, velocity and caliber of load are interrelated in the downstream direction.

In the present state of knowledge it is not possible to substitute as simple and readily understandable a concept as the one to which we have been accustomed. A few relations which may be necessary and useful for further understanding can be stated now, but formulation of a complete picture must await further work.

In streams carrying primarily coarse gravel, it is safe to assume that bed roughness is governed primarily by particle size. Downstream, where the bed is composed of fine sand, silt, or clay, bed roughness is materially affected by the configuration of the bed in the form of transitory riffles, waves, and dunes.

The vertical-velocity profile is dependent in part on the bed roughness. Roughness affects the rate at which velocity decreases as one approaches the bed—that is, the shear in the moving water close to the bed. This shear is intimately related to the production of turbulence, which in turn is responsible for the movement of bed particles.

Movement of bed particles is accomplished by the energy of turbulent eddies immediately adjacent to the particles. A similar principle applies to the quarrying of large blocks of bedrock exposed in the stream bed and to the movement of large boulders or coarse gravel on the bed. On the basis of keen observation, Matthes (1947) described how kolk, a form of macroturbulence consisting of strong intermittent vortices, is a primary mechanism for plucking and for movement of large bed material. Vortical motion is probably, in most cases, a phenomenon arising from shear stress.

Therefore, a change in roughness, through its effect on shear, would alter the competence even without any concurrent change in mean velocity. Similarly, competence may be changed without necessarily altering slope.

If roughness decreased downstream, shear would tend to decrease downstream. This implies a downvalley decrease in competence.

The relation between shear and bed roughness and between these same factors and competence and load are beyond the scope of this paper. The importance of these relationships has been brought out by the many studies of bed load movement in which the grain size and shear factors appear prominently. The present paper merely calls attention to a tendency for certain shear functions to decrease downstream. The tendency for particle size to decrease downstream is well known. There is implied a downstream decrease of competence but any definitive discussion of this aspect requires further detailed study.

From the foregoing analysis it is clear that we may no longer consider stream adjustment in terms of a downstream decrease in velocity. It is equally important to recognize, however, that the introduction of the concept of decreasing shear downstream does not mean that we may simply substitute decreasing shear or roughness for decreasing velocity in our old concept of stream adjustment. As can be seen in the Manning equation, a change of roughness, n, requires a change in the interrelations of velocity, depth, and slope in order that the equation be satisfied. Moreover, if stream adjustment is considered in terms of the eight variables, discharge, width, depth, velocity, slope, roughness, load, and caliber of load, as was done by Leopold and Maddock (1953), then there are not one but eight simultaneous equations involved in the description of the interrelations.

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CONTENTS

AN EXPERIMENTAL TEST OF VISUAL COMPARISON TECHNIQUE IN ESTIMATING TWO DIMENSIONAL SPHERICITY AND ROUNDNESS OF QUARTZ VEINS
Melvin A. Rosenfeld and John C. Griffiths 553

FURTHER DATA ON THE SYSTEM Na_2O—CaO—SiO_2
E. R. Segnit 586

RUIN-MARBLE: A STYLOLITIC FORMATION?
B. M. Shaub 602

DOWNSTREAM CHANGE OF VELOCITY IN RIVERS
Luna B. Leopold 606

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